# MATHCOUNTS ${ }^{\circ}$ 

## 2023 State Competition

Target Round Problems 1 \& 2
Name
School $\qquad$
Chapter $\qquad$

## DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

| Problem 1 | Problem 2 | Scorer's Initials |
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$\qquad$ An arithmetic operation $(+,-, \times$, or $\div$ ) is written in each blank in the expression 4 $\qquad$ 3 evaluated?
2. $\qquad$ The point P has coordinates $(m, n)$ where $m$ and $n$ are integers. Suppose that P is reflected across the $x$-axis to obtain the point Q . Then the point Q is reflected across the $y$-axis to obtain the point R. Given that the area of triangle PQR is 80 units $^{2}$, what is the least possible value of $|m+n|$ ?


## DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

| Problem 3 | Problem 4 | Scorer's Initials |
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3.
4. $\qquad$ $\mathrm{cm}^{2}$ In the figure shown, the center of circle O is located on side PQ of square PQST . Given that $\mathrm{PO}=2 \mathrm{~cm}, \mathrm{RS}=4 \mathrm{~cm}$, and $\mathrm{ST}=6 \mathrm{~cm}$, what is the area of the shaded region? Express your answer in terms of $\pi$.



## DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

| Problem 5 | Problem 6 | Scorer's Initials |
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5. $\qquad$ The set of integers $\{1,3,9,11, x\}$ has the same mean and median. What is the sum of the possible values of $x$ ?
6. $\qquad$ ways

Spencer writes a two-digit number in each of the blanks in the sentence shown to make the sentence a true statement. Both digits of each number are drawn from the set $\{1,4,7\}$, and Spencer is allowed to repeat digits and write the same number more than once. In how many different ways could he fill in the three blanks?

The number $\qquad$ is the arithmetic mean of $\qquad$ and $\qquad$ .


## DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

| Problem 7 | Problem 8 | Scorer's Initials |
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7.
$\mathrm{m}^{2}$ Right triangle DEF has one side of length 2023 meters and is similar to the right triangle shown. What is the area of triangle DEF if all of its sides have integer lengths?

8. $\qquad$ The sequence $a_{1}, a_{2}, \ldots, a_{6}$ is a nonconstant geometric sequence of positive integers in which each term is less than 2023. What is the greatest possible value of $a_{1}+a_{2}+\cdots+a_{6}$ ?

