MATHCOUNTS®

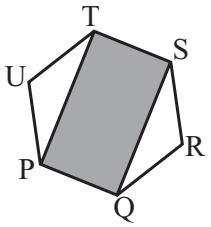
2023 State Competition

Countdown Round Problems 1–80

This booklet contains problems to be used in the Countdown Round.

1. _____ (band members) The members of a marching band are placed in a circle and numbered consecutively. If band member 8 is standing directly opposite band member 27, how many band members are there?

2. _____ In the figure shown, PQRSTU is a regular hexagon. What is the ratio of the area of quadrilateral PQST to the area of hexagon PQRSTU? Express your answer as a common fraction.



3. _____ (plates) Kate's catering service offers a date cake. A recipe to make 12 plates of date cake requires 18 dates. How many whole date cake plates can caterer Kate make with 88 dates?

4. _____ (numbers) A *yearly number* is a number that contains the sequence "2023". For example, 312023 is a yearly number but 120023 is not. How many numbers that have seven digits and a nonzero millions digit are yearly numbers?

5. _____ (mi/h) Charlie skied from Any Town to Big City, a distance of 12 miles, in 75 minutes. What was Charlie's average speed, in miles per hour? Express your answer as a decimal to the nearest tenth.

6. _____ The first number that appears in both the arithmetic sequences $\{-8, -5, -2, 1, \ldots\}$ and $\{-3, 2, 7, 12, \ldots\}$ is 7. What is the seventh number that appears in both sequences?

7. _____ (meters) The dimensions of a rectangular prism, in meters, are all integers. If its length is twice its width, and its volume is 4200 m^3, what is the greatest possible width of the prism?

8. _____ A $2 \times 2 \times 2$ cube is removed from each corner of an $8 \times 8 \times 8$ cube. What fraction of the original cube's volume remains? Express your answer as a common fraction.

9. _____ What is the value of $\sqrt{3^2 + 4^2 + 12^2}$?

10. _____ What is the value of the expression $\dfrac{3^{n+3} - 3(3^n)}{9(3^n)}$ when $n = 2022$? Express your answer as a common fraction.

11. _____ When $x = 7$, $x^2 + 11x + c = 0$. What is the value of c?

12. _____ (segments) A circle has 9 points evenly spaced around the circumference. How many different segments can be formed by joining a pair of these points?

13. _____ The area of Circle 1 is three times the area of Circle 2. What is the ratio of the radius of Circle 1 to the radius of Circle 2? Express your answer in simplest radical form.

14. _____ If $7k + 4j = 35$ and $5k + 8j = 25$, what is the value of $k + j$?

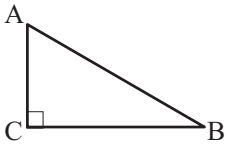
15. _____ What is the value x if $20x + 23 = 2023$?

16. _____ (digits) How many digits are there in $50^6 \times 4^3$ when it is written out in base 10?

17. _____ (%) A two-by-four is a piece of lumber of varying length that is cut to be two inches deep and four inches wide. Then it is sanded down until smooth. The finished size is $1\frac{3}{4}$ inches by $3\frac{3}{4}$ inches. What percent of the volume of the wood is removed in the sanding process? Express your answer to the nearest percent.

18. _____ (positive integers) How many positive integers n satisfy the relation $2^{300} < n^{100} < 3^{200}$?

19. _____ (degrees) In the right triangle shown, the ratio of AB to BC is the same as the ratio of AB to AC. What is the degree measure of angle B?



20. _____ (ft^3) A 12-ft deep, 40-ft wide and 25-ft long swimming pool is being filled with water. If 1425 cubic feet of the pool's volume needs to be left empty, how many cubic feet of water is used to fill the pool?

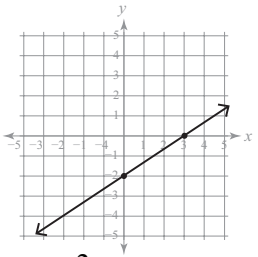
21. _____ What is the mean of the following numbers: 0, 0, 1, 2, 3, 4, 5, 10, 10, 10? Express your answer as a decimal to the nearest tenth.

22. _____ Rachel cuts a straight length of ribbon at a random place on the ribbon. What is the probability she cuts it in such a way that one of the pieces is at least twice as long as the other? Express your answer as a common fraction.

23. _____ What is the value of the expression $\frac{2^{2024} - 2^{2022}}{2^{2023} - 2^{2021}}$?

24. _____ (inches) What is the circumference of a circle whose area is 225π in^2? Express your answer in terms of π.

25. _____ The figure shows the graph of the line $x = ay + b$. What is the value of $a + b$? Express your answer as a common fraction.



26. _____ What is the least possible value of x that satisfies the equation $\frac{x}{2} = \frac{3}{x+1}$?

27. _____ (years old) Noor is the youngest of five sisters. The average of their ages, which are all different whole numbers, is 12 years. What is the oldest that Noor can be, in years?

28. _____ (cm) The combined volume of a sphere of radius 2 cm and a cylinder of radius 2 cm and height 2 cm equals the volume of a cone of radius 1 cm and height h. What is the value of h?

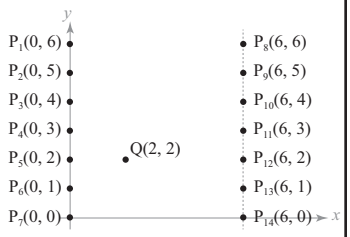
29. _____ (%) A $3 \times 3 \times 3$ wooden cube has three faces painted blue, and then the cube is cut into 27 unit cubes. What percentage of the combined surface area of the 27 unit cubes is blue? Express your answer to the nearest whole percent.

30. _____ (ounces) In water, the ratio of the weight of oxygen to the weight of hydrogen is approximately 8:1. Given that 16 ounces equals a pound, how many ounces of oxygen would be in 1 pound of water? Express your answer as a mixed number.

31. _____ What is the value of the expression $2023^2 - 1977^2$?

32. _____ The image of P(2, 3) reflected across the line $x = -1$ is P′. The image of P′ reflected across the line $y = x$ is P″. What is the sum of the x- and y-coordinates of P″?

33. _____ Consider the graph shown here. Suppose that two distinct points are randomly selected from the set $\{P_1, P_2, ..., P_{14}\}$ and a line is drawn through the two points. What is the probability that the line passes through point Q(2, 2)? Express your answer as a common fraction.



34. _____ (laps) Johnny swims 8 laps in the pool every Monday and every Tuesday. He swims 12 laps in the pool every Saturday. Given that January 1, 2022 was a Saturday, and January has 31 days, how many laps did Johnny swim in January 2022?

35. _____ When A (3, 8) is rotated clockwise 90 degrees about the origin, the image is A′(x, y). What is the value of $x + y$?

36. _____ What is the value of the expression $0.\overline{23} + \frac{2}{3}$? Express your answer as a common fraction.

37. _____ What is the sum of all of the integer solutions x to the equation $(x - 4)^{4x - 8} = 1$?

38. _____ (playlists) At a party, the DJ will play three different cumbias and three different merengues. If each song plays once, and the cumbias and the merengues must alternate in the playlist, what is the number of possible playlists?

39. _____ For how many positive integers n is $\frac{2022}{n-1}$ an integer?

40. _____ What is the value of the expression $2^3 \times 4^2 \times 16^{-1}$?

41. _____ A right triangle has side lengths 5 cm, 12 cm and 13 cm. An altitude is drawn from the right angle to the opposite side, dividing the triangle into two smaller triangles. Let A be the area of the smaller of those two triangles and let B be the area of the larger triangle, in square centimeters. What is the ratio of A to B? Express your answer as a common fraction.

42. _____ If $(x + y)^2 = 47$ and $(x - y)^2 = 11$, then what is the value of xy?

43. _____ When the result of multiplying 20^4 by sixty thousand is expressed in scientific notation, the result is $a \times 10^b$. What is the value of a? Express your answer as a decimal to the nearest tenth.

44. _____ (miles) On January 1, Jane walks 27 miles. On January 2, Jane walks 9 miles. On January 3, Jane walks 3 miles. She continues in this way, so that each day she walks $\frac{1}{3}$ as far as the day before. There are 31 days in January. How many miles does Jane walk in January? Express your answer as a decimal to the nearest tenth.

45. _____ A number is increased by 50%, then the result is increased by 40%. If the final number is 23.1 greater than the original, what is the original number?

46. _____ What is the least positive integer with 8 odd positive divisors and 16 even positive divisors?

47. _____ If $\frac{2}{x} < \frac{x}{7}$, what is the least possible positive integer value of x?

48. _____ If the average of 35, 45 and x is equal to five more than twice x, what is the value of x?

49. _____ When $y = 1.5$, what is the value of $\frac{(y+1)^4 - (y+1)^3}{(y+1)^2}$? Express your answer as a decimal to the nearest hundredth.

50. _____ (cups) Miranda measures out $\frac{5}{8}$ of a cup of quinoa, which will provide 1 serving for each of her 6 dinner guests. However, 2 more guests show up late. How many additional cups of quinoa does Miranda need to measure out for the extra guests, if they want 1 serving each? Express your answer as a common fraction.

51. _____ (percent) A business suit has been discounted by 30%. A coupon offers 25% off the already reduced price. If the final price after sales tax is 56.7% of the original price, then what is the percent rate of sales tax?

52. _____ What is the value of $\frac{5!}{4!} \times \frac{3!}{2!}$?

53. _____ If $3^{6x+2} = \frac{1}{81}$, what is the value of 2^{3-4x}?

54. _____ Point $(m, -103)$ lies on the line whose equation is $x - 4y = -40$. What is the value of m?

55. _____ A square is inscribed in a circle which is then inscribed in a larger square. What is the ratio of the area of the smaller square to the area of the larger square?

56. _____ If the remainder when $6n$ is divided by 7 is 5, what is the remainder when n is divided by 7?

57. _____ If $3m + 12n = 51$ and $7m + 3n = -6$, what is the value of $n - m$?

58. _____ (joules) An incandescent bulb which uses 100 joules of energy per second is replaced with an LED which uses 17 joules per second. How many fewer joules does the LED bulb use than the incandescent bulb over a period of one hour?

59. _____ (times) A square pyramid and a square prism have the same height and the same length of side of the bases. How many times would James need to fill the pyramid with sand in order to fill the prism?

60. _____ (pictures) A book has 163 pages. On the second page there is a picture, and there is a picture on every sixth page thereafter. How many pictures are in the book?

61. _____ (adult tickets) A total of two hundred adult and student tickets were sold to the spring play. Adults were charged \$8 each, and children were charged \$5 each. If there was a total of \$1375 collected, then how many adult tickets were sold?

62. _____ (mi/h) Beth entered a speed skating competition where she completed the 6-mile course in 12 minutes. What was her average speed, in miles per hour?

63. _____ (globbers) In mudderball, a flibber is worth 6 points and a globber is worth 3 points. Suppose a team scored 210 points and made 8 more flibbers than globbers. How many globbers did the team make?

64. _____ (seats) In a stadium, there are 20 seats in the first row. Every row after the first row has 4 more seats than the preceding row. If the stadium has 50 rows, how many seats are in the stadium?

65. _____ (dollars) Tess buys 37 pairs of gloves for \$2.97 each. To the nearest dollar, how much money does she spend?

66. _____ (hexagons) The sum of the degree measures of the interior angles of 9 triangles, 5 rectangles, and 3 pentagons is equal to the sum of the interior angles of how many hexagons?

67. _____ What is the sum of all values of x that satisfy the equation $x^4 - 5x^2 + 4 = 4$?

68. _____ (years old) Mr. Rasmussen is 84 years old. He has a son and a grandson. When Mr. Rasmussen was as old as his son is now, his grandson was 2 years old. If the sum of the ages of Mr. Rasmussen and his grandson is 114, how old is Mr. Rasmussen's son?

69. _____ What is the absolute difference between the geometric and arithmetic means of 8 and 50?

70. _____ (programs) Jenny enters a bird calling contest in which she chooses an ordered program listing of three different birds from the following choices: the American black duck, which goes *quack*; the black skimmer, which goes *kawp*; the dusky grouse, which goes *kwa*; and the yellow-billed cuckoo, which goes *kowp*. How many possible programs exist which have Jenny say *kwa*?

71. _____ If the points $(-1, 2)$, $(-10, 19)$ and $(b, -32)$ lie on the same line, then what is the value of b?

72. _____ Nine consecutive integers have a sum of 144. What is the product of the least and the greatest of the nine integers?

73. _____ (numbers) Ryan makes a list of odd integers from 1 to 100 that only have an even number of positive factors. How many numbers are in his list?

74. _____ (degrees) What is the degree measure of the largest angle of a hexagon if the angle measures form an arithmetic sequence and the smallest angle measures 80 degrees?

75. _____ (locations) Kelly the kangaroo starts at the origin of a coordinate plane. In any given hop, she can hop exactly one unit up or one unit to the right. At how many distinct locations can she end up after 100 hops?

76. _____ What is the sum in base 10 of 44_5 and 32_5?

77. _____ What is the least positive integer n that satisfies the equation $(2022 - n)^2 = (n - 2022)^2$?

78. _____ What is the value of the expression $\frac{2^{10} - 8^3}{16^2 + 4^3}$? Express your answer as a common fraction.

79. _____ Tickets to a musical were sold either at full price or at half price. If the half-price tickets brought in 50% more money than the full-price tickets, what fraction of the tickets were sold at half price? Express your answer as a common fraction.

80. _____ (feet) The area of Jack's kitchen is 135 ft^2, and its width is three-fifths of its length. What is the sum of its length and width, in feet?