

MATHCOUNTS®

2023 CHAPTER COMPETITION Sprint Round Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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1. \$ _____



What is the total cost of 3 jars of peanut butter priced \$5 each and 5 jars of jelly priced \$3 each?

2. _____

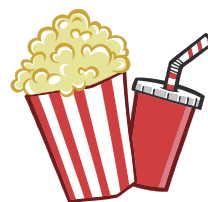
What is the absolute difference between 2^5 and 5^2 ?

3. _____

What is the value of the expression $\frac{1}{5} + \frac{11}{15}$? Express your answer as a common fraction.

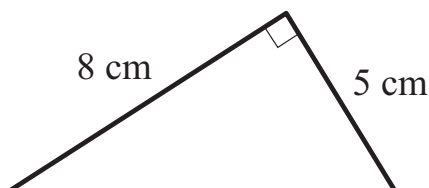
4. \$ _____

Bob has \$50 in his pocket when he sets off for the movie theatre. After he pays \$10 for a ticket, \$8 for popcorn and \$7 for a soda, how much money does Bob have left in his pocket?



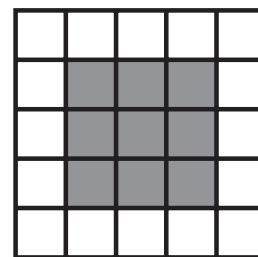
5. _____ cm^2

What is the area of the right triangle shown with legs of lengths 5 cm and 8 cm?



6. _____ %

The figure shows a large square divided into 25 congruent squares. What percent of the figure is shaded?



7. _____

On the number line shown, the number n is one-third of the way from 0 to 6. What is the value of n ?

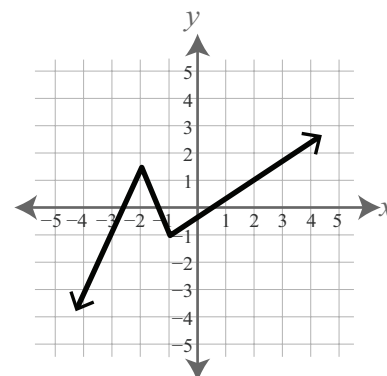


8. _____

What is the value of $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of 240?

9. _____

For the function graphed here, what is the greatest value of x for which $y = -1$?



10. _____

If $4n + 1 = 250$, what is the nearest integer to the value of n ?

11. _____ What is the value of the expression $\sqrt{\frac{1}{4}} + \sqrt{\frac{1}{9}} + \sqrt{\frac{1}{16}}$? Express your answer as a common fraction.

12. _____ The sum of two numbers is 9 and their absolute difference is 3. What is their product?

13. _____ fish Of the fish in Ari's aquarium, $\frac{1}{2}$ are red, $\frac{1}{4}$ are blue and $\frac{1}{8}$ are black. The remaining 4 fish are yellow. Given that all of Ari's fish are a single color, how many fish are in Ari's aquarium?

14. _____ marbles A bag is filled with 100 marbles each colored red, white or blue. The table shows the results when Cia randomly draws 10 marbles. Based on this data, how many of the marbles in the bag are expected to be red?

Red	White	Blue

15. _____ tea parties Margaret holds tea parties every Tuesday afternoon for the purpose of using her collection of 100 teacups. If she invites n people, she will use $n + 1$ teacups: one for each invited guest and one for herself. If she has already had 24 tea parties, each with two guests, how many tea parties with three guests should she host to ensure each teacup is used exactly once?



16. _____ : _____ a.m.



Evan gets stuck in an elevator. At 12:11 a.m., the elevator repair company dispatches a technician who is 75 miles away. The technician drives at an average speed of 50 mi/h, and after arriving, takes 10 minutes to enter the building, and then an additional 7 minutes to unlock the elevator. At what time is Evan released from the elevator?

17. _____

If the median of the data set $\{x + 2, x + 3, x - 4, x - 1, x + 1\}$ is 6, what is the value of x ?

18. _____

Let x , y and z be positive integers with $x < y < z$. If the mean of x , y and z is 99, what is the greatest possible value of z ?

19. _____

Which digit should replace E in the units place of the number 1,234,56E so that the number is divisible by 9?

20. _____ minutes

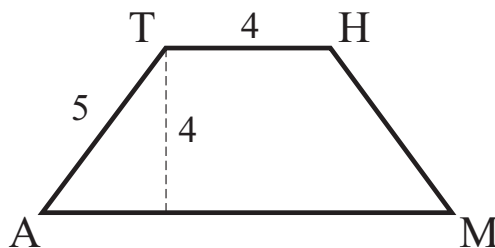
During P.E. class, twelve students take turns playing an 8-player game. If class lasts 1 hour and each student plays the same amount of time, what is the maximum number of minutes each student can actively be playing the game?



21. _____ lengths Triangle ABC has $AB = 10$ and $BC = 6$. How many different integer lengths are possible for side AC?

22. _____ Consider the geometric sequence $a_1 = 1, a_2 = 2, a_3 = 4, a_4 = 8, \dots$. What is the average of the first 10 terms of this sequence? Express your answer as a decimal to the nearest tenth.

23. _____ cm Isosceles trapezoid MATH, shown here, has height 4 cm. If $AT = HM = 5$ cm and $TH = 4$ cm, what is the perimeter of trapezoid MATH?



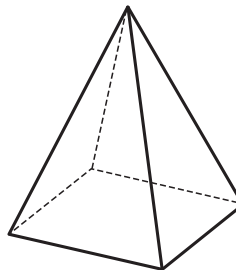
24. _____ If $x + 2y = 9$ and $5x + 4y = -4$, what is the value of $8x + 10y$?

25. _____ cupcakes Grace made over four dozen cupcakes. If she makes packages of 2 cupcakes, then there is 1 left over. Packaging in groups of 3 cupcakes leaves 2 left over, and packaging in groups of 4 cupcakes leaves 3 left over. What is the fewest number of cupcakes Grace could have made?



26. _____ What is the value of $\frac{\sqrt{45} + 2\sqrt{15}}{\sqrt{128} + 2\sqrt{24}}$? Express your answer as a common fraction in simplest radical form.

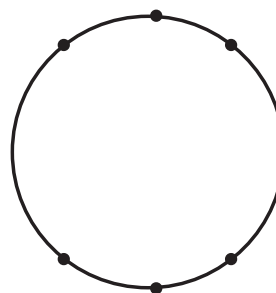
27. _____ cm^3 Each face of a right square pyramid has a perimeter of 24 cm. What is the volume of the pyramid? Express your answer in simplest radical form.



28. _____ If each of a , b and c is the cube of some nonzero integer, what is the product of all of the different possible values of $\frac{abc}{|abc|} + \frac{ab}{|ab|} + \frac{ac}{|ac|} + \frac{bc}{|bc|} + \frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|}$?

29. _____ cm The area of a rectangle is 32 cm^2 . What is the least possible integer perimeter of the rectangle?

30. _____ Consider the six points on the circle shown. If George draws line segments connecting pairs of points so that each point is connected to exactly two other points, what is the probability that the resulting figure is a convex hexagon? Express your answer as a common fraction.



Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

Geometric figures may not be drawn to scale and lengths of geometric figures should be assumed to be measured in “units” unless otherwise stated.

All answers must be expressed in simplest form. A “common fraction” is to be considered a fraction in the form $\pm \frac{a}{b}$, where a and b are natural numbers and $\text{GCF}(a, b) = 1$. In some cases the term “common fraction” is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified “mixed number” (“mixed numeral,” “mixed fraction”) is to be considered a fraction in the form $\pm N \frac{a}{b}$, where N , a and b are natural numbers, $a < b$ and $\text{GCF}(a, b) = 1$. Examples:

Problem: What is $8 \div 12$ expressed as a common fraction?

Answer: $\frac{2}{3}$

Unacceptable: $\frac{4}{6}$

Problem: What is $12 \div 8$ expressed as a common fraction?

Answer: $\frac{3}{2}$

Unacceptable: $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π ?

Answer: $\frac{1+2\pi}{8}$

Problem: What is $20 \div 12$ expressed as a mixed number?

Answer: $1\frac{2}{3}$

Unacceptable: $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$

Unacceptable: $3\frac{1}{2}$, $\frac{1}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form?

Answer: $5\sqrt{3}$

Unacceptable: $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., “How many dollars...,” “How much will it cost...,” “What is the amount of interest...”) should be expressed in the form (\$) $a.bc$ or $a.bc$ (dollars), where a is an integer and b and c are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c both are zero, in which case they both may be omitted. Answers in the form (\$) $a.bc$ or $a.bc$ (dollars) should be rounded to the nearest cent unless otherwise specified. Examples:

Acceptable Forms: 2.35, 0.38, .38, 5.00, 5

Unacceptable: 4.9, 8.0

Do not make approximations for numbers (e.g., π , $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the “rounding” a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:

Problem: What is 6895 expressed in scientific notation?

Answer: 6.895×10^3

Problem: What is 40,000 expressed in scientific notation?

Answer: 4×10^4 or 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted.

Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.