MATHCOUNTS® Problem of the Week Archive

Holiday Packages – December 12, 2022

Problems & Solutions

Speedy Delivery charges $20.00 to send a 1-pound package via two-day service and an additional $2.00 for every 4 ounces of package weight over 1 pound. How much will it cost to ship a package that weighs 3.75 pounds via Speedy Delivery's two-day service?

*Since 1 pound is included in the initial $20.00, Cornelius' mom will be charged the $2.00 per 4-ounce overage rate for 3.75 pounds – 1 pound = 2.75 pounds. The 2.75 pounds is equivalent to 2.75 pounds × 16 ounces/pound = 44 ounces and will result in 44 ounces × $2.00/4 ounces = $22.00 in overage charges. Thus, the total cost to ship the package via Speedy Delivery's two-day service will be $20.00 + $22.00 = $**42.00**.*

Cornelius bought a spherical ball to give to his friend during the holiday gift exchange. The ball came in a cubic box with a volume of 1331 in^3. If the ball fits snugly in the box so that every side of the box just touches the ball, how many cubic inches of space inside of the box are not filled by the ball? Express your answer as a decimal to the nearest tenth.

*Since the ball fits snugly in the box, with the ball touching all interior sides of the box, the diameter of the ball is equal to the length of the edge of the box. The volume of a cube is equivalent to the length of its edge cubed, thus the length of the edge of the box is $\sqrt[3]{1331} = 11$ inches. That means that the radius of the ball is 11 inches ÷ 2 = 5.5 inches. The volume of a sphere is $V = 4/3 \times \pi \times r^3$, thus the volume of the ball is $4/3 \times \pi \times 5.5^3 \approx 696.9\ in^3$. Thus, the space inside the box not filled by the ball is $1331\ in^3 - 696.9\ in^3$ = **634.1** in^3.*

If eight congruent spherical balls are placed in the 1331 in^3 box instead, what is the largest possible radius, in inches, of each of the balls? Express your answer as a decimal to the nearest hundredth.

If eight of the largest possible congruent balls are placed in the box, there will be 4 balls on the bottom of the box and four stacked on top of each of the balls. That means that each ball will have a diameter that is equal in length to 1/2 of the length of the edge of the box. We know the edge of the box is 11 inches from the previous problem, so the diameter of each of the balls is 11 ÷ 2 = 5.5 inches. Thus, the radius of each of the balls is 5.5 ÷ 2 = ***2.75*** *inches.*

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