

MATHCOUNTS[®] Problem of the Week Archive

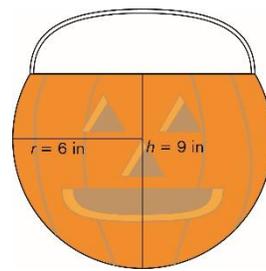
Trick-or-Treat – October 31, 2022

Problems & Solutions

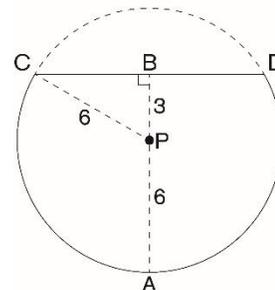
Ariel, Ollie and Eden are going trick-or-treating for Halloween, but first their parents are taking them to a costume store to pick out costumes. The store has 9 different costume options available – witch, pirate, cat, superhero, princess, vampire, fire fighter, zombie, clown. In how many different ways can Ariel, Ollie and Eden each select one of these costumes, with no two of them wearing the same costume?

There are 9 options for the first costume. Since no two of them will wear the same costume, this leaves 8 options for the second costume and 7 options for the third costume. In the end, there will be $9 \times 8 \times 7 = 504$ different ways for them to each select one of the 9 costumes, with no two of them wearing the same costume.

To carry their candy, Ariel, Ollie and Eden each have a bucket that looks like a pumpkin. It is completely spherical in shape with a straight slice across the top of the bucket that creates a circular opening for dropping in candy. If the radius of the bucket is 6 inches and the height, measured from the base of the bucket to the open top, is 9 inches, what is the area of the circular opening of the bucket, in square inches? Express your answer in terms of π .



The spherical bucket has radius 6 inches, and the circular opening is created by a slice through the sphere 9 inches from its base. Since both measurements are with respect to the sphere's center, let's simplify the problem by looking at it in two dimensions. We'll focus on a circle with points on the sphere whose center coincides with the center of the sphere, sometimes called a "great circle". Consider the circle shown, with radius 6 inches and segment AB drawn through center P perpendicular to chord CD, such that AB = 9 inches. Notice that the chord CD is a diameter of the circular opening of the bucket. To calculate the area, we need the radius of the opening, or CB. Since AP = 6 inches, BP = 9 - 6 = 3 inches. If we draw radius CP, we create 30-60-90 right triangle PBC. We can conclude that CB = $3\sqrt{3}$ inches. Therefore, the circular opening of the bucket has area $\pi(3\sqrt{3})^2 = 27\pi$ in².



Last year, after Ariel, Ollie and Eden went trick-or-treating, they combined all their candy together and sorted it. They found that $\frac{1}{3}$ was chocolate candy, $\frac{1}{4}$ was hard candy, $\frac{1}{5}$ was candy corn and the rest were fruit chews. If they had a 180 pieces of candy total, how many pieces were fruit chews?

If $\frac{1}{3}$ of their candy was chocolate, $\frac{1}{4}$ was hard candy and $\frac{1}{5}$ was candy corn, then $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$ of their candy was one of these three types. The fraction of the candy that was fruit chews would have been $1 - \frac{47}{60} = \frac{13}{60}$. If they had 180 pieces of candy, then the number of pieces that were fruit chews would be $180 \times (\frac{13}{60}) = 39$ pieces.

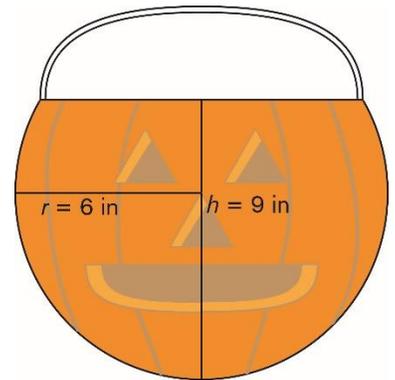
MATHCOUNTS[®] Problem of the Week Archive

Trick-or-Treat – October 31, 2022

Problems

Ariel, Ollie and Eden are going trick-or-treating for Halloween, but first their parents are taking them to a costume store to pick out costumes. The store has 9 different costume options available – witch, pirate, cat, superhero, princess, vampire, fire fighter, zombie, clown. In how many different ways can Ariel, Ollie and Eden each select one of these costumes, with no two of them wearing the same costume?

To carry their candy, Ariel, Ollie and Eden each have a bucket that looks like a pumpkin. It is completely spherical in shape with a straight slice across the top of the bucket that creates a circular opening for dropping in candy. If the radius of the bucket is 6 inches and the height, measured from the base of the bucket to the open top, is 9 inches, what is the area of the circular opening of the bucket, in square inches? Express your answer in terms of π .



Last year, after Ariel, Ollie and Eden went trick-or-treating, they combined all their candy together and sorted it. They found that $\frac{1}{3}$ was chocolate candy, $\frac{1}{4}$ was hard candy, $\frac{1}{5}$ was candy corn and the rest were fruit chews. If they had a 180 pieces of candy total, how many pieces were fruit chews?