

# MATHCOUNTS<sup>®</sup> Problem of the Week Archive

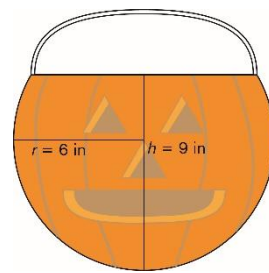
## Trick-or-Treat – October 31, 2022

### Problems & Solutions

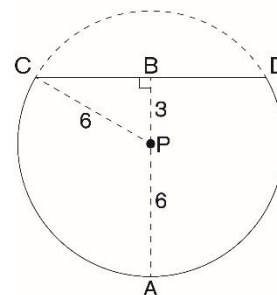
Ariel, Ollie and Eden are going trick-or-treating for Halloween, but first their parents are taking them to a costume store to pick out costumes. The store has 9 different costume options available – witch, pirate, cat, superhero, princess, vampire, fire fighter, zombie, clown. In how many different ways can Ariel, Ollie and Eden each select one of these costumes, with no two of them wearing the same costume?

*There are 9 options for the first costume. Since no two of them will wear the same costume, this leaves 8 options for the second costume and 7 options for the third costume. In the end, there will be  $9 \times 8 \times 7 = 504$  different ways for them to each select one of the 9 costumes, with no two of them wearing the same costume.*

To carry their candy, Ariel, Ollie and Eden each have a bucket that looks like a pumpkin. It is completely spherical in shape with a straight slice across the top of the bucket that creates a circular opening for dropping in candy. If the radius of the bucket is 6 inches and the height, measured from the base of the bucket to the open top, is 9 inches, what is the area of the circular opening of the bucket, in square inches? Express your answer in terms of  $\pi$ .



*The spherical bucket has radius 6 inches, and the circular opening is created by a slice through the sphere 9 inches from its base. Since both measurements are with respect to the sphere's center, let's simplify the problem by looking at it in two dimensions. We'll focus on a circle with points on the sphere whose center coincides with the center of the sphere, sometimes called a "great circle". Consider the circle shown, with radius 6 inches and segment AB drawn through center P perpendicular to chord CD, such that AB = 9 inches. Notice that the chord CD is a diameter of the circular opening of the bucket. To calculate the area, we need the radius of the opening, or CB. Since AP = 6 inches, BP = 9 - 6 = 3 inches. If we draw radius CP, we create 30-60-90 right triangle PBC. We can conclude that CB =  $3\sqrt{3}$  inches. Therefore, the circular opening of the bucket has area  $\pi(3\sqrt{3})^2 = 27\pi$  in<sup>2</sup>.*



Last year, after Ariel, Ollie and Eden went trick-or-treating, they combined all their candy together and sorted it. They found that  $\frac{1}{3}$  was chocolate candy,  $\frac{1}{4}$  was hard candy,  $\frac{1}{5}$  was candy corn and the rest were fruit chews. If they had a 180 pieces of candy total, how many pieces were fruit chews?

*If  $\frac{1}{3}$  of their candy was chocolate,  $\frac{1}{4}$  was hard candy and  $\frac{1}{5}$  was candy corn, then  $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$  of their candy was one of these three types. The fraction of the candy that was fruit chews would have been  $1 - \frac{47}{60} = \frac{13}{60}$ . If they had 180 pieces of candy, then the number of pieces that were fruit chews would be  $180 \times (\frac{13}{60}) = 39$  pieces.*

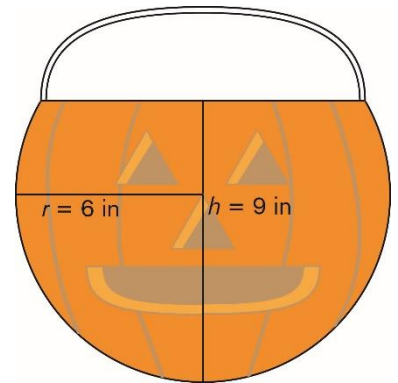
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