

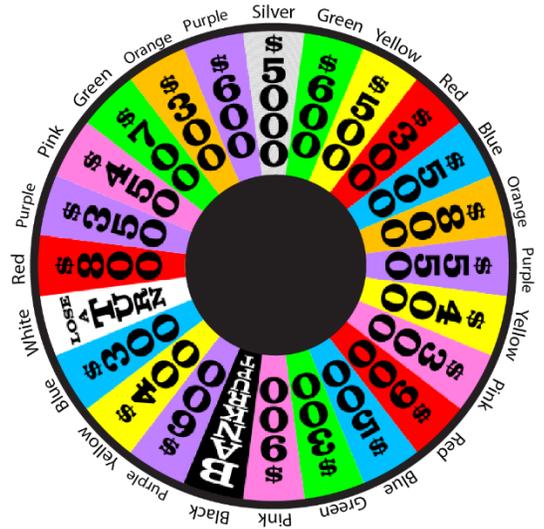
MATHCOUNTS® Problem of the Week Archive

Winning Big! – August 8, 2022

Problems & Solutions

On the game show Wheel of Fortune®, contestants spin a wheel, like the one shown here, to determine how much money will be won for solving the puzzle.

What is the probability that a contestant spins the wheel and it stops on a space that either is not purple or is worth more than \$500? Express your answer as a common fraction.



The probability that, once spun, the wheel stops on a space that is not purple is $20/24 = 5/6$. The probability that the wheel stops on a space that is worth more than \$500 is $10/24 = 5/12$. Seven of the spaces that are worth more than \$500 are also not purple and we need to account for these duplicates. Thus, the probability of the wheel stopping on a space that either is not purple or a space worth more than \$500 is $5/6 + 5/12 - 7/24 = (20 + 10 - 7)/24 = 23/24$.

In how many ways with multiple spins can a contestant spin a sum of \$1200 with the wheel stopping on spaces of a single color and stopping on a space no more than twice? (The order of the spins matters.)

First, we see that with each of the colors, it is possible to get a sum of \$1200 in multiple spins. However, to get a sum of \$1200 in a single color with blue or orange would require the wheel to stop on a space more than twice. So, let's examine ways to get a sum of \$1200 with the wheel stopping on green, red, pink, yellow and purple spaces.

Green

$\$600 \times 2$: Since there is 1 green \$600, this can happen in 1 way.

$\$300 \times 2 + \600 : Since there is 1 green \$300 and 1 green \$600, this can happen in 3 ways.

Red

$\$300 + \900 : Since there is 1 red \$300 and 1 red \$900, this can happen in 2 ways.

Pink

$\$300 + \900 : Since there is 1 pink \$300 and 1 pink \$900, this can happen in 2 ways.

$\$450 \times 2 + \300 : Since there is 1 pink \$450 and 1 pink \$300, this can happen in 3 ways.

Yellow

$\$400 \times 2 + \400 : Since there are 2 yellow $\$400$ sections, let's call one A and one B. The possible spin options to get $\$1200$, then, are ABB, BAB, BBA, BAA, ABA, AAB. Thus, this can happen in 6 ways.

Purple

$\$600 \times 2$: Since there are 2 purple spaces with $\$600$, this can happen in 4 ways.

That means a contestant can spin a sum of $\$1200$ with the wheel stopping on spaces of a single color and stopping on a space no more than twice in $1 + 3 + 2 + 2 + 3 + 6 + 4 = \mathbf{21}$ ways.

What is the probability that a contestant spins the wheel three times and it stops on a different color each time with the same dollar amount? Express your answer as a common fraction.

The only amount that appears on the wheel in 3 or more different colors is $\$300$. There is 1 red, 1 pink, 1 green, 1 blue and 1 orange space labeled $\$300$. From these, we see that there are ${}_5C_3 = 10$ combinations of 3 different colors, which can each occur in $3!$ different ways. That's a total of $10 \times 6 = 60$ ways. There are $24^3 = 13,824$ results when the wheel is spun three times. That means the probability of the wheel stopping on three different colors with the same dollar amount in three spins is $60/13,824 = \mathbf{5/1152}$. You can also consider that on the first spin, $5/24$ spaces will work. On the second spin, $4/24$ spaces will work. And on the third spin, only $3/24$ spaces work. The probability is $5/24 \times 4/24 \times 3/24 = 5/2 \times 1/24 \times 1/24 = \mathbf{5/1152}$.

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