MATHCOUNTS® Problem of the Week Archive

Making Sense of Numbers – July 25, 2022

Problems & Solutions

Here are a few problems to keep your number sense sharp during the summer months.

What is the greatest positive three-digit integer that is divisible by 5, 7 and 9?

We know that any three-digit number that is divisible by 5, 7 and 9 is divisible by 5 × 7 × 9 = 315. The largest three-digit multiple of 315 is 315 × 3 = ***945****.*

What is the greatest possible product of a pair of two-digit integers, composed of the digits 8, 6, 4 and 2 if each digit is used exactly once?

Using each of the digits 8, 6, 4 and 2 exactly once to make a pair of two-digit integers, we will achieve the greatest product of these two integers if one number has a tens digit of 8 and the other has a tens digit of 6. So, our products are 84 × 62 = 5208 and 82 × 64 = 5248. The greatest product is ***5248****.*

A proper divisor of a number is a divisor of the number that is not the number itself. What is the smallest positive integer that is less than the sum of its positive proper divisors?

Right away, we can eliminate any prime number since its only factors are 1 and itself. The sums of the proper divisors of positive integers beginning with 4 are listed below.

4: 1 + 2 = 3
6: 1 + 2 + 3 = 6
8: 1 + 2 + 4 = 7
9: 1 + 3 = 4
10: 1 + 2 + 5 = 8
12: 1 + 2 + 3 + 4 + 6 = 16

So, ***12*** *is the smallest positive integer that is less than the sum of its proper divisors.*

For how many positive four-digit integers is the sum of its digits equal to the product of its digits?

The only four digits that have the same sum and product are 1, 1, 2 and 4, since 1 + 1 + 2 + 4 = 1 × 1 × 2 × 4 = 8. There are 4!/2! = 4 × 3 = ***12*** *positive four-digit integers containing these digits.*

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