MATHCOUNTS® Problem of the Week Archive

Some math with JULY – July 11, 2022

Problems & Solutions

When the letters of the alphabet are assigned their integer values (A = 1, B = 2, C = 3, …, Z = 26), the word-product for JULY is the product of the letter-values for each of the letters in JULY. When the square root of JULY's word-product is put into its simplest radical form of $a\sqrt{b}$, where b has no perfect-square factors greater than 1, what is the value of b?

We don't have to find the word-product for JULY; we just need to know the factors that go into it. We see J = 10, U = 21, L = 12 and Y = 25, so the word-product is 10 × 21 × 12 × 25, and we need to take the square root of this value eventually. Factoring everything, we see we have $2^3 \times 3^2 \times 5^3 \times 7$. When we take the square root and pull out the factors used twice, we will have 2 × 5 × 7 = ***70*** *still under the radical.*

We can also take the word-product of sequences of letters; they don't necessarily have to be a word. What sequence of letters has the same word-product as JULY, but (1) does not contain any of the letters in JULY, (2) contains the letter N, (3) does not contain any of its letters more than once and (4) has its letters in alphabetical order?

Again, the prime factorization of JULY's word-product helps: $2^3 \times 3^2 \times 5^3 \times 7$. So, we know we have $N \times 2^2 \times 3^2 \times 5^3$. Since we can't use any letters in JULY, we must get our three factors of 5 from the letters with values of 5, 15 and 20. These letters are E, O and T. Now, we have N × E × O × T × 3. Fortunately, C = 3 is still available, and we have ***CENOT****.*

Similar to a word-product, a word-sum is the sum of the letter-values for each of the letters of a sequence. The values of the letters of a particular four-letter sequence form an arithmetic sequence and this sequence has the same word-sum as JULY. What is the least possible value of a letter in this four-letter sequence?

The word-sum of JULY is 10 + 21 + 12 + 25 = 68. Notice the average value of the four letters is 68 ÷ 4 = 17. If our new four-letter sequence has a sum of 68 and the four values form an arithmetic sequence, then we know 17 is halfway between the second and third values. This means our second and third values could be 16 & 18, 15 & 19, 14 & 20, etc. If the second and third values are 16 & 18, then we would have the values 14, 16, 18, 20. If the second and third values are 15 & 19, then we would have the values 11, 15, 19, 23. If the second and third values are 14 & 20, then we would have the values 8, 14, 20, 26. This is as far apart as we can go for the second and third values, since we can't have a value higher than 26. Of these three options for our sequence, we see that the least possible letter-value is ***8****.*

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