MATHCOUNTS® Problem of the Week Archive

Three Sons and a Daughter – June 20, 2022

Problems & Solutions

Archer has three sons named Baker, Carver and Decker. Archer's age is five times the sum of Carver's and Decker's ages. The sum of Baker's and Decker's ages is half Archer's age. If Baker's age is five times Carver's age, what is the ratio of the ages of Carver and Decker? Express your answer as a common fraction.

*Let's first write some equations to represent the information we're given. Let the ages of Archer, Baker, Carver and Decker be represented by a, b, c and d, respectively. We are told that Archer's age is five times the sum of Carver's and Decker's ages. That yields the equation a = 5 × (c + d). Next, we are told that half Archer's age is equal to the sum of the ages of Baker and Decker. That results in the equation (1/2) × a = b + d → a = 2 × (b + d). Finally, we know that Baker's age is five times Carver's age. So, we have b = 5c. We are asked to determine the ratio of the ages of Carver and Decker, which can be represented as c/d. Since we have two equations that represent Archer's age, we can set them equal to each other to get 5 × (c + d) = 2 × (b + d). Simplifying, we have 5c + 5d = 2b + 2d → 5c + 3d = 2b. We can substitute 5c for b in the equation 5c + 3d = 2b to get 5c + 3d = 2 × 5c → 5c + 3d = 10c. Simplifying, we see that the ratio of Carver's age to Decker's age is 3d = 5c → c/d = **3/5**.*

In addition to the information given in the previous problem, we now are told the product of Carver's age and Decker's age is equal to Baker's age. What is the sum of the ages of Baker, Carver and Decker?

*We now know that b = cd, and from the previous problem we know that b = 5c. Therefore, 5c = cd → d = 5. Also recall from the previous problem that c/d = 3/5 → 5c = 3d. Substituting 5 for d yields 5c = 3 × 5 → 5c = 15 → c = 3. Finally, we know that b = cd, and substituting 3 for c and 5 for d yields b = 3 × 5 = 15. Now we know that Baker is 15 years old, Carver is 3 years old and Decker is 5 years old. Therefore, the sum of the ages of Baker, Carver and Decker is 15 + 3 + 5 = **23** years.*

Archer's first-born child is a daughter whose name is Tailor. If Tailor is 18 years old, how old was Archer when she was born?

*Recall from the first problem that a = 5 × (c + d). Since we determined that c = 3 and d = 5 in the previous problem, we can substitute to get a = 5 × (3 + 5) = 5 × 8 = 40. That means Archer is currently 40 years old. Eighteen years ago, when Tailor was born, Archer was 40 − 18 = **22** years old.*

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In addition to the information given in the previous problem, we now are told the product of Carver's age and Decker's age is equal to Baker's age. What is the sum of the ages of Baker, Carver and Decker?

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