MATHCOUNTS® Problem of the Week Archive

Counting Down to the National Competition – May 2, 2022

Problems & Solutions

The 2022 Raytheon Technologies MATHCOUNTS National Competition is coming up next weekend, and there has been lots of preparation going on behind the scenes! For example, T-shirts had to be ordered well in advance for all 240 students and 60 coaches. If T-shirts were ordered for every student and coach, as well as enough for another 132 individual orders, and the ratio of Small:Medium:Large:XL ordered was 5:11:6:2, how many medium shirts were ordered?

The total number of T-shirts ordered is 240 + 60 + 132 = 432. From the ratio 5:11:6:2, the ratio of medium:total would be 11/(5 + 11 + 6 + 2) = 11/24. Therefore, to find the number of medium T-shirts that were ordered, we can set up and solve a proportion by cross-multiplying: M/432 = 11/24 → 24M = 432 × 11 → 24M = 4752 → M = 198. So, ***198*** *T-shirts were ordered in size medium.*

Many volunteers give of their time to help with local and state competitions throughout the year, and the National Competition is no different! Ribbons are put on the name badges of attendees to designate the different ways they are helping the program. Suppose the details of one volunteer list (which shows who should get which ribbon(s)) are as follows:

1. There are only 4 types of ribbons on the list: Volunteer, Scorer, Monitor and Timer (A person can have more than one ribbon attached to his/her name badge.)
2. Everyone has a Volunteer ribbon.
3. Only 3 people have exactly 3 ribbons.
4. There are 7 people with Monitor ribbons and 12 people with Scorer ribbons.
5. Only 2 people have exactly 4 ribbons.
6. There are an equal amount of Timers who are also Monitors as Timers who are not Monitors.
7. Exactly 5 people have Scorer and Monitor ribbons.
8. There are twice as many people that have exactly one ribbon as people who have multiple ribbons.

How many volunteers are on this list?

Let's approach this by first finding the number of people with 4 ribbons, then 3, then 2, then 1.

4-ribbon volunteers*: The first thing to notice is condition #5, which tells us only 2 people have exactly 4 ribbons. So, there are 2 4-ribbon volunteers.*

3-ribbon volunteers*: The next clue is condition #7 because it says that 5 people have Scorer and Monitor ribbons. Two of those people were already accounted for in the 4-ribbon category. Therefore, the other 3 of them have Monitor, Scorer and Volunteer ribbons (because condition #2 says everyone has a Volunteer ribbon). There are 3 3-ribbon volunteers. (Notice that condition #3 says only 3 people have exactly 3 ribbons, so these are those 3 people.)*

2-ribbon volunteers*: Everyone left has either a Volunteer ribbon + 1 other ribbon or only a Volunteer ribbon. From condition #4, we know that there are 7 Monitors, five of whom have already been accounted for, so there are 7 – 5 = 2 people that have both a Monitor and a Volunteer ribbon. Also from condition #4, we know that there are 12 Scorers, five of whom have already been accounted for, so there*

are 12 – 5 = <u>7</u> people that have both a Scorer and a Volunteer ribbon. Finally, working with condition #6, there were only 2 Timers that were also Monitors (in the 4-ribbon category), so there must be <u>2</u> that have both a Timer and a Volunteer ribbon. Therefore, there are 2 + 7 + 2 = <u>11</u> volunteers with 2 ribbons.

***1-ribbon volunteers**: We have found a total of 2 + 3 + 11 = 16 volunteers with more than one ribbon. From condition #8, there are twice as many people that have exactly one ribbon as people who have multiple ribbons. Therefore, there are 16 × 2 = <u>32</u> 1-ribbon volunteers.*

*Thus, there are a total of 16 + 32 = **48** volunteers on the list.*

Let's say that in the final round of the Countdown Round, the winner is the first competitor to get three questions correct. Assume no more than 5 questions are used and every question asked is answered correctly by one of the competitors. How many different ways can the final round play out such that Competitor A wins the round? (Example I: Competitor A could get the first 3 questions correct (AAA), Example II: Competitor B could get only the second question correct (ABAA).)

*If only 3 questions are asked, then there is only <u>1</u> way the round can play out (AAA). If only 4 questions are asked, then there are <u>3</u> possibilities: BAAA, ABAA or AABA. If 5 questions are asked, we know that competitor A must have gotten the last question correct, so the possibilities will all be in the form _ _ _ _ A. We can calculate ${}_4C_2$ to determine how many ways there are to place 2 Bs in 4 spots: 4!/(2!2!) = <u>6</u> ways (BBAAA, BABAA, BAABA, ABBAA, ABABA and AABBA). Therefore, there are 1 + 3 + 6 = **10** ways the final round can play out under the guidelines provided.*

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