MATHCOUNTS® Problem of the Week Archive

2022 State Competition – April 11, 2022

Problems & Solutions

Now that the 2022 State Competitions have passed, here are some problems the State level Mathletes solved during the competition!

2022 State Competition, Sprint Round, #6

While driving, Carl notices that his odometer reads 25,952 miles, which happens to be a palindrome. He thought this was pretty rare, but 2.5 hours later, his odometer reads as the next palindrome number of miles. What was Carl's average speed during those 2.5 hours?

*The first palindrome is at 25952. Any 5-digit palindrome starting with 25 must end in 52, and we can have an arbitrary digit in the middle. Getting the next palindrome in the 25 thousands would require the next digit after 9 for the middle digit, but there is no such digit. Thus, we must go to the 26 thousands, and the palindrome must end in 62. The least middle digit is 0. The next palindrome after 25952 is 26062, with a separation of 110. Therefore, the travel is 110 miles in 2.5 hours for an average speed of (110 mi)/(2.5 h) = **44** mi/h.*

2022 State Competition, Sprint Round, #14

Brian has earned 65%, 80% and 92% on his three pre-final exams. These exams are not weighed equally: the lowest counts for only 20% of his overall grade, while the other two count for 25% each. If the final exam is the remainder of the overall grade and there are no opportunities for extra credit, what is the highest grade Brian can earn in the class? Express your answer to the nearest whole percent.

*Let's express the weights as decimal numbers rather than percentages: 20% = 0.20 to be applied to the lowest pre-final exam score, which is 65%; 25% = 0.25 to be applied to the other pre-final exam scores, 80% and 92%. The sum of the weights must be 100% = 1.00, so the final exam must have a weight of 1.00 – (0.20 + 0.25 + 0.25) = 0.30. We are to assume the maximum possible score of 100% for the final exam. The weighted average is 0.20 × 65 + 0.25 × 80 + 0.25 × 92 + 0.30 × 100 = 13 + 20 + 23 + 30 = **86**%.*

2022 State Competition, Target Round, #3

The table shows the percent of families in Mathville that have 0, 1, 2, 3 and 4 or more children. If there are a total of 10,250 families, how many are there with no children?

Number of children	0	1	2	3	4+
% of total families	x	20	18	10	6

*Because the total percentage of families is 100%, the childless families must account for x = 100 – 20 – 18 – 10 – 6 = 46% = 0.46. The number of families with no children, then, is 0.46 × 10,250 = **4715** families.*

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