MATHCOUNTS® Problem of the Week Archive

2022 Chapter Competition – March 28, 2022

Problems & Solutions

Here is a look at some of the problems our Mathletes solved on this year's Chapter Competition in February!
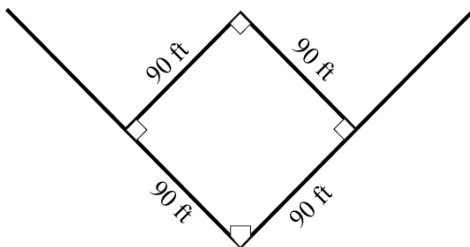
2022 Chapter Competition, Sprint Round, #11

A nonzero number of goats, each with four legs, and a nonzero number of chickens, each with two legs, are living on a farm. Between all the animals, there are 3/8 as many heads as there are legs. Assuming that all animals have all of their limbs, what is the fewest possible number of chickens on the farm?

*Because there are 3/8 as many heads as there are legs, there are 8/3 as many legs as heads, so the average number of legs per head is 2 2/3, which is ((2 2/3) – 2)/(4 – 2) = (2/3)/2 = 1/3 of the way from 2 [# of legs per chicken] to 4 [# of legs per goat]. Therefore, 1/3 of the animals are goats, and 2/3 are chickens, so the goat-to-chicken ratio is 1:2. Thus, the minimum number of animals is 1 + 2 = 3 animals, meaning there is 1 goat and **2** chickens.*

2022 Chapter Competition, Target Round, #5

A baseball diamond is a square with 90-foot sides. An acre of land is a region containing 43,560 square feet. What percent of an acre is enclosed within a baseball diamond? Express your answer as a decimal to the nearest tenth.

*The area of a baseball diamond is 90^2 = 8100 square feet. So, a baseball diamond covers (8100/43,560) × 100 ≈ **18.6**% of an acre.*

2022 Chapter Competition, Team Round, #6

For her 16th birthday, Kate receives a book. On each page whose digits sum to 16, there is one word of a secret birthday message written in the margin. If the book is 280 pages, how many words are in the secret birthday message?

For a page number 100h + 10t + u ≤ 280, h, t and u represent the hundreds, tens and units or ones digits, respectively, each at least 0 and at most 9, subject to further constraints, two of which are h + t + u = 16 and h ≤ 2. When h = 0, t + u = 16, so t can range from 7 up to 9, while u ranges from 9 down to 7 – this makes 3 options. When h = 1, t + u = 15, so t can range from 6 up to 9, while u ranges from 9 down to 6 – this makes 4 options. Careful now: When h = 2, t + u = 14, so t can range from 5 up to only 7 [due to a

*page number being at most 280], while u ranges from 9 down to 7 – this makes 3 options. Therefore, the total number of words written is 3 + 4 + 3 = **10** words.*
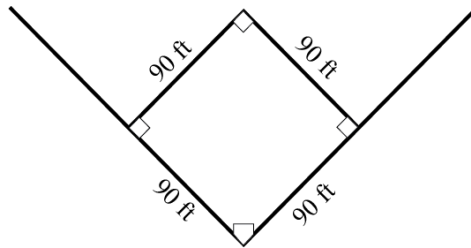
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