

# MATHCOUNTS® Problem of the Week Archive

## St. Patrick's Day Math – March 21, 2022

### Problems & Solutions

Last Thursday, March 17 was St. Patrick's Day when, typically, more people than usual wear some article of green clothing. Patrick has a leprechaun hat that looks like a tall, green top hat. The main portion of the hat is a cylinder that is 8 inches tall and has a 3-inch radius. He filled this portion of the hat with chocolates wrapped to resemble small gold coins to pass out to all his friends. One bag of these chocolate "coins" takes up about  $20 \text{ in}^3$  of space. What is the minimum number of bags of chocolate that Patrick must get to completely fill his hat?

*The formula for the volume of a cylinder is  $\pi \times r^2 \times h$ . We know that this hat has radius 3 inches and height 8 inches, so the area is  $\pi \times 3^2 \times 8 \approx 226.2 \text{ in}^3$ . The chocolates from one bag take up  $20 \text{ in}^3$  of space. Since  $226.2 \div 20 = 11.31$ , Patrick will need to get **12** bags of chocolates.*

Margaret painted a mural for St. Patrick's Day and mixed her own green paint. She mixed 3 parts yellow paint with 2 parts blue paint to create green paint. Maureen wanted to also paint a mural with the same green color, but she currently has 8 cups of green paint that is a mixture of 40% yellow paint and 60% blue paint. To get the same shade of green as Margaret, how many cups of yellow paint must she add to her mixture?

*Maureen currently has  $0.40 \times 8 = 3.2$  cups of yellow paint and  $0.60 \times 8 = 4.8$  cups of blue paint in her mixture. We know that she needs more yellow paint, so she will eventually have  $3.2 + x$  cups of yellow paint and 4.8 cups of blue paint, and we want this to be a ratio of 3:2. So, we can set up the proportion  $(3.2 + x)/4.8 = 3/2$ . Cross-multiplying, we get  $6.4 + 2x = 14.4$ . Subtracting 6.4 from both sides leads to  $2x = 8$  and  $x = 4$ . Therefore, Maureen needs to add **4** cups of yellow paint to the current mixture to get Margaret's shade of green.*

Let's now take a look at the word GREEN. There are not too many real words that can be made from the letters in GREEN. For this problem, though, let's see how many ways we can arrange the five letters in the word GREEN, even if they don't form real words. But let's also add a restriction: any arrangement must keep the two Es together. How many such arrangements are there?

*If we're keeping the two Es together, then we might as well just replace EE with, let's say, a P. So, now we just need to figure out how many arrangements of the letters G, R, P and N exist. There are four letters, so there are  $4! = \mathbf{24}$  ways they can be arranged.*

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### ***Problems***

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