MATHCOUNTS® Problem of the Week Archive

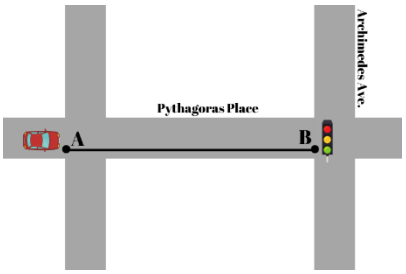
Calling All Future Engineers! – February 14, 2022

Problems & Solutions

In honor of Engineers Week, we're posting a throwback to a sampling of previous years' Future Engineers Problems of the Day. Join us next week for all new engineering-themed Problems of the Day for #EWeek2022!

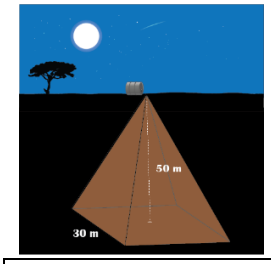
Civil Engineering (2020)

Say the distance from point A to point B is 1320 feet. The driver in the red car looks up and sees the light at the intersection up ahead (Pythagoras Place and Archimedes Ave) just turned green. If that light only stays green for 25 seconds, what is the minimum average speed, in miles per hour, she must drive to get from point A to point B while that light is still green?



We can use the formula Distance = Rate × Time. Knowing the distance from point A to point B is 1320 feet, we can write 1320 = r × 25. Dividing both sides of the equation by 25 gives us a rate of 52.8 ft/s. However, we are asked for the average speed in miles per hour. We know that there are 3600 seconds in 1 hour, so we can calculate the rate as 52.8 × 3600 = 190,080 ft/h. Knowing there are 5280 feet in 1 mile, 190,080 ÷ 5280 = ***36*** *mi/h.*

Environmental Engineering (2021)



A drum of dry-cleaning solvent, a toxic chemical, is dumped and seeps into the soil, contaminating the groundwater below. This space where contaminants have spread is called a ***plume***. Exactly one year after the solvent was dumped, the plume forms a square pyramid underground that has spread to a depth of 50 m and 30 m across at its base edge. What is the volume of the plume after one year?

The volume of a square pyramid is calculated using the formula $V = a^2(h/3)$, where a = the length of a base edge and h = the height of the pyramid. So, the volume of this plume is $30^2(50/3) = (900 \times 50)/3 =$ ***15,000*** *m^3.*

Software Engineering (2021)

When someone sends a text message over a communications network, receivers detect if any errors occurred when the message was transmitted. A ***checksum*** is one common algorithm to do this: it identifies *if* an error occurred, but does not specify *which* error(s) and cannot catch every kind of error. For example, if numbers were assigned to every letter of the alphabet (A = 1, B = 2, ..., Z = 26), and someone sent a message that said OK, the values of these letters would be O = 15 and K = 11. The checksum algorithm would verify the message being received had a sum of 15 + 11 = 26 and, if the sum were *not* 26, would identify an error in the transmission. However, if the message above came through as KO, the sum would still equal 26, meaning the checksum would not catch this specific error.

Vanessa texts René the message BAD. Assigning the same numbers to the letters in the example above, and assuming all three-letter arrangements are equally likely, what is the probability that René receives a three-letter message that has an error the checksum does *not* catch? Express your answer as a common fraction.

Checksum does not catch errors in messages that have the correct three letters in the incorrect order. There are 3 × 2 × 1 = 6 arrangements of the letters BAD, of which only 1 is the correct message, so there are 6 – 1 = 5 ways that the message could have the correct three letters with an error that checksum does not catch. The sum of B + A + D = 2 + 1 + 4 = 7, so we must also consider any three-letter arrangements with incorrect letters that also sum to 7. For example, E + A + A = 5 + 1 + 1 = 7 as well, but this is obviously the wrong message. In total, there are 9 ways that checksum could miss an error with incorrect letters involved (EAA, AEA, AAE, BBC, BCB, CBB, ACC, CAC, CCA), as the sum of each of these arrangements is also 7. So, checksum would miss a total of 5 + 9 = 14 errors. There is a total of 26 × 26 × 26 = 17,576 possible three-letter arrangements, so the probability that you receive your friend's message with an error that checksum does not catch is 14/17,576 = ***7/8788****.*

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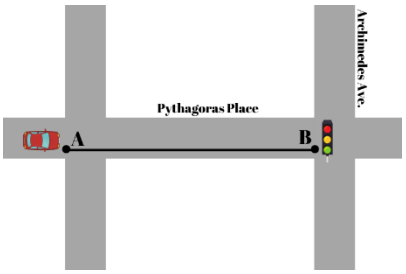

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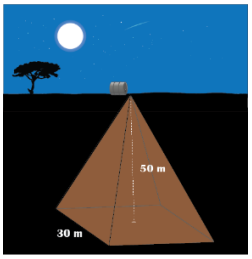

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