

MATHCOUNTS® Problem of the Week Archive

Sold Out – January 31, 2022

Problems & Solutions

The Agnesi High School auditorium has exactly 26 rows of seats. The rows are labeled, in order, from the front of the auditorium to the back from A through Z. There are 8 seats in the row A. Each row after the first row has two more seats than the previous row. There are 10 seats in row B, 12 seats in row C and so on. How many seats are there in row Z?

*The number of seats in each row form the arithmetic sequence 8, 10, 12, 14, 16, ..., where $a = 8$, the common difference is $d = 2$, and the n th term of the sequence can be determined using the formula $a + d(n - 1)$. Thus, the 26th term of the sequence is $8 + 2(25) = 8 + 50 = 58$. Therefore, there are **58** seats in Row Z.*

What is the total number of seats in the Agnesi High School auditorium?

From the previous problem, we know that the number of seats in each row form the arithmetic sequence with terms $a, a + d, a + 2d, a + 3d, \dots, a + 24d, a + 25d$, where $a = 8$ and $d = 2$. It follows that the sum of the terms is $a + a + d + a + 2d + a + 3d + \dots + a + 24d + a + 25d = 26a + d(1 + 2 + 3 + \dots + 24 + 25)$. The sum $1 + 2 + 3 + \dots + 24 + 25 = (1 + 25)(25/2) = 26(25/2) = 13(25) = 325$. So, the total number of seats in the auditorium is $26a + 325d = 26(8) + 325(2) = 208 + 650 = \mathbf{858}$ seats.

Agnesi High School collected \$2860 from ticket sales for the winter concert. A ticket was sold for every seat in the auditorium, resulting in a sold-out concert. If the price of an adult ticket was three times the price of a student ticket, and twice as many student tickets were sold as adult tickets, what was the price of a student ticket?

*Let x and y represent the number of student and adult tickets sold, respectively. We are told that twice as many student tickets were sold as adult tickets. We also are told that a ticket was sold for every seat, and from the previous problems, we know that there are 858 seats in the auditorium. So, we have the equations: $x = 2y$ and $x + y = 858$. Substituting $2y$ for x in the second equation, we have $2y + y = 858 \rightarrow 3y = 858 \rightarrow y = 286$, which means $x = 2(286) = 572$. So, 286 adult tickets and 572 student tickets were sold for the concert. Now let m and n represent the price of a student and adult ticket, respectively. We are told that a total of \$2860 was collected from selling student and adult tickets, and that the price of an adult ticket was three times the price of a student ticket. Based on this information, we can write the following equations: $572m + 286n = 2860$ and $n = 3m$. Substituting $3m$ for n in the first equation, we have $572m + 286(3m) = 2860 \rightarrow 572m + 858m = 2860 \rightarrow 1430m = 2860 \rightarrow m = 2$. Therefore, the price of a student ticket was **\$2.00**.*

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