MATHCOUNTS® Problem of the Week Archive

Best of 2021! – December 27, 2021

Problems & Solutions

While there were many problems written and solved for MATHCOUNTS in 2021, below is a selection of some of the best of the year!

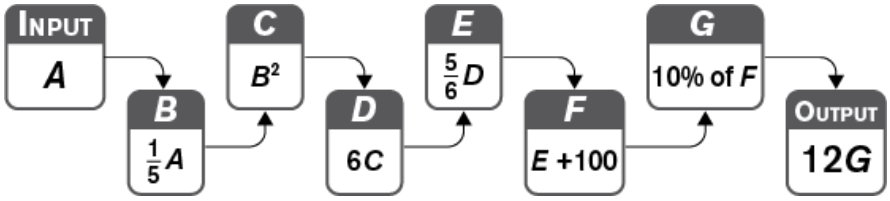
2020-2021 School Handbook, #113

A fair coin is flipped 5 times. What is the probability that no two consecutive flips have the same result? Express your answer as a common fraction.

*There are $2^5 = 32$ outcomes for 5 flips of a fair coin. The only 2 outcomes without consecutive heads or consecutive tails are HTHTH and THTHT. Therefore, the desired probability is 2/32 = **1/16**. Alternatively, in order for no two consecutive coin flips to result in both heads or both tails, the 2nd flip must be different from the 1st flip (P = 1/2), the 3rd flip must be different from the 2nd flip (P = 1/2), the 4th flip must be different from the 3rd flip (P = 1/2) and the 5th flip must be different from the 4th flip (P = 1/2). Thus, the probability of this occurring is 1/2 × 1/2 × 1/2 × 1/2 = **1/16**.*

2020-2021 School Handbook, #142

Devon wrote a program that takes a positive integer *A* as an input and performs a series of operations, each time assigning the result to a new variable, as shown. If the output of the program is 144, what was the value of the input *A*?



*We know the output, so we can work in reverse to determine the value of the input. We start with 12G = 144, so G = 144 ÷ 12 = 12. Next, we have (1/10)F = 12, so F = 12 × 10 = 120. Next, E + 100 = 120, so E = 120 – 100 = 20. Then, (5/6)D = 20, so D = 20 × 6/5 = 24. Next, 6C = 24, so C = 24 ÷ 6 = 4. Next, $B^2 = 4$, so B = √4 = ±2. Since we are told that A is a positive integer, it must be that (1/5)A = 2, and A = 2 × 5 = **10**.*

2021 Chapter Sprint Round, #15

What is the value of $(1.6 + 5)^2 – 1.6^2 – 5^2$?

*Squaring the binomial of the expression $(1.6 + 5)^2 – 1.6^2 – 5^2$ gives us $1.6^2 + 1.6 \times 5 + 1.6 \times 5 + 5^2 – 1.6^2 – 5^2$. Some terms are opposites, so we can further simplify: $\boxed{1.6^2} + 1.6 \times 5 + 1.6 \times 5 \boxed{+ 5^2} \boxed{– 1.6^2} \boxed{– 5^2} = 2 \times 1.6 \times 5 = 1.6 \times 10 =$ **16**.*

2021 Chapter Invitational Sprint Round, #22

Julie is packing marbles into boxes. She needs to pack away 815 marbles. She has boxes that can hold 10, 25, 50 or 100 marbles. If Julie can use at most 5 boxes of each size and must fill each box she uses, what is the minimum number of boxes she needs to pack all her marbles?

All of the box sizes but one, the 10-marble box, holds a multiple of 25 marbles, so we need to decrement the total marble count of 815 successively by 10 until we reach a multiple of 25. We want the next odd multiple of 25 down from 815 to minimize the use of inefficient 10-marble boxes. That multiple is 775, with 4 of the 10-marble boxes to get there. Similarly, we need to decrement by 25 successively until we get to the next multiple of 50 down from 775, which is 750, requiring 1 of the 25-marble boxes. Again, similarly, we need to decrement by 50 successively until we get to the next multiple of 100 down from 750, which is 700, requiring 1 of the 50-marble boxes. That leaves 700 marbles and boxes of 100, so we need 7 such boxes. However, we have only 5 such boxes, so we must convert the need for 2 of them to 4 boxes of 50 marbles each. Putting all this together means we have 5 boxes of 100, 5 boxes of 50, 1 box of 25 and 4 boxes of 10, and no count exceeds 5. Therefore, the minimum total needed box count is 5 + 5 + 1 + 4 = ***15*** *boxes.*

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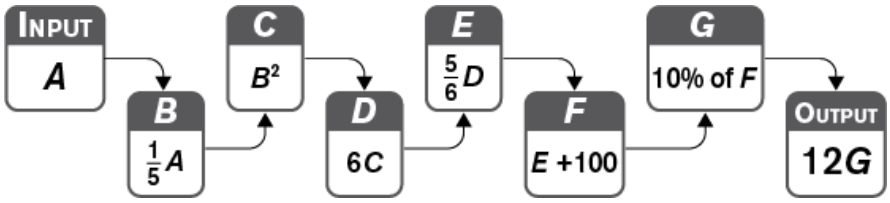

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