

# MATHCOUNTS<sup>®</sup> Problem of the Week Archive

**Happy Hat Day! – January 10, 2022**

## Problems & Solutions

January 15<sup>th</sup> is Hat Day, so put on your wackiest hat and solve this week's problems!



Ariana loves hats. Her favorite top hat is an 18-inch tall right circular cylinder with a brim that goes all the way around the opening for her head. The top surface of the hat as well as the brim are covered in solid yellow fabric, while only the lateral surface of the cylinder is covered with patterned fabric, as shown. If the cylinder has a diameter of 7.5 inches, what is the lateral surface area of the cylinder? Express your answer in terms of  $\pi$ .

*The lateral surface area of a cylinder is the product of the cylinder's height and the circumference of its base. We are told that this cylinder has height 18 inches and base diameter 7.5 inches. The circumference of the cylinder, then, is  $7.5\pi$  inches, and it follows that the lateral surface area is  $7.5\pi \times 18 = 135\pi$  in<sup>2</sup>.*

If the brim on Ariana's favorite top hat is 4.5 inches wide, what is the circumference of the outer edge of the brim? Express your answer in terms of  $\pi$ .

*Since the brim of Ariana's hat is 4.5 inches wide, the outer edge of the brim has a diameter of  $7.5 + 9 = 16.5$  inches. Therefore, the outer edge of the brim of Ariana's hat has a circumference of  $16.5\pi$  inches.*

What is the area of the top surface of the brim on Ariana's favorite top hat? Express your answer in terms of  $\pi$ .

*Since the outer edge of the brim has a diameter of 16.5 inches, it follows that its radius is  $16.5/2 = 8.25$  inches. The area of a circle with a radius of 8.25 inches is  $8.25^2\pi = 68.0625\pi$  in<sup>2</sup>. To determine the area of the top surface of the brim, we must subtract the area of the circular opening for Ariana's head. This opening has a radius of  $7.5/2 = 3.75$  inches, and an area of  $3.75^2\pi = 14.0625\pi$  in<sup>2</sup>. Thus, the area of the top surface of the brim of Ariana's hat is  $68.0625\pi - 14.0625\pi = 54\pi$  in<sup>2</sup>.*

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## ***Problems***

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If the brim on Ariana's favorite top hat is 4.5 inches wide, what is the circumference of the outer edge of the brim? Express your answer in terms of  $\pi$ .

What is the area of the top surface of the brim on Ariana's favorite top hat? Express your answer in terms of  $\pi$ .