

# MATHCOUNTS®

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## 2022 STATE COMPETITION Team Round Problems 1–10

School \_\_\_\_\_  
Chapter \_\_\_\_\_  
Team Members \_\_\_\_\_, Captain  
\_\_\_\_\_  
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### DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials



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1. \$ \_\_\_\_\_ On Monday, Hama started work at 9:33 a.m. and ended work at 5:20 p.m., and she took a break from 12:18 p.m. to 12:41 p.m. If Hama was paid \$15 per hour of work after taxes (and she was not paid for her break time), how much money was Hama paid on Monday? Express your answer to the nearest dollar.

2. \_\_\_\_\_ turns Three players are sitting at a round table playing a dice game with these rules. Each player starts with three tokens. On each turn, the player rolls one regular six-sided die and does one of the following actions based on the number rolled:

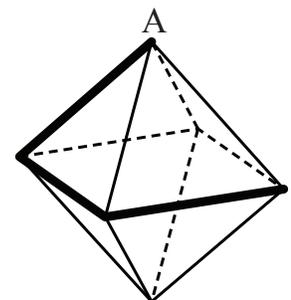
Number	Action
1	Place one token in the middle of the table.
2	Give one token to the player on your right.
3	Give one token to the player on your left.
4, 5 or 6	Keep your tokens.

After each turn, play passes to the player on the left. The game ends as soon as there is only one player with tokens left. What is the fewest number of total turns that can be taken before the game is over?

3. \_\_\_\_\_ goals per game In a 50-game lacrosse season, there were 24 games in which Hayley scored at least 1 goal, there were 8 games in which she scored at least 2 goals, and there were 2 games in which she scored 3 goals. If Hayley never scored 4 goals or more in one game, what was the average number of goals per game Hayley scored during the season? Express your answer as a decimal to the nearest hundredth.

4. \_\_\_\_\_ numbers Amos counts out loud by six beginning with 0. Sarah counts out loud by four also beginning with 0. Between 2000 and 3000 inclusive, how many of the numbers do both Sarah and Amos say out loud?

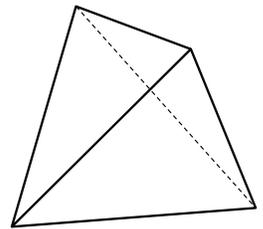
5. \_\_\_\_\_ Consider the regular octahedron as shown. Each edge of the octahedron has a length of 1. An ant starts at the vertex A and crawls a total distance of 3 units along the edges of the octahedron. Any time the ant reaches a vertex of the octahedron, it randomly chooses an edge to next crawl on that is different from the edge it just left. One such path the ant may take is shown. What is the probability that the ant will end up a distance greater than 1 from its starting point A? Express your answer as a common fraction.



6. 3-partitions If  $n$  is a positive integer, a 3-partition of  $n$  is a list of powers of 3, arranged in order from greatest to least, whose sum is  $n$ . For example, there are three 3-partitions of 6, namely 3-3, 3-1-1-1 and 1-1-1-1-1-1. How many different 3-partitions of 29 are there?

7. \_\_\_\_\_ Consecutive positive even integers are sorted into 100 groups. Group 1 includes 2, 4 and 6. Group 2 includes 8, 10, 12 and 14. Group 3 includes 16, 18, 20, 22 and 24. Each successive group has one additional number in it than the previous group. What is the sum of the numbers in the 100th group?

8. ways Each face of a regular tetrahedron, like the one shown, is painted red, blue, green or yellow. How many unique ways exist to paint the tetrahedron in this manner? Two colorings of the tetrahedron are considered the same if and only if one can be rotated to obtain the other.



9. \_\_\_\_\_ A fair coin is tossed repeatedly until either heads comes up three times in a row or tails comes up three times in a row. What is the probability that the coin will be tossed more than 10 times? Express your answer as a common fraction.

10. cm<sup>2</sup> Consider the equilateral triangle ABC with sides of length  $8\sqrt{3}$  cm. A point in the interior of ABC is said to be “special” if it is a distance of 3 cm from one side of the triangle and a distance of 7 cm from another side. Consider the convex polygon whose vertices consist of the special points. What is the area of this polygon? Express your answer as a decimal to the nearest tenth.

