MATHCOUNTS®

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2022 STATE COMPETITION
Sprint Round Problems 1–30

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HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

Raytheon Technologies

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03-S22SPR

1. _____ What is the slope of the line containing the points (3, 4) and (5, 12)?

2. _____ °F This chart shows the weather conditions in Madison, WI from January 11 through January 15. What was the mean high temperature for those five days in Madison? Express your answer as a decimal to the nearest tenth.

	Monday Jan. 11	Tuesday Jan. 12	Wednesday Jan. 13	Thursday Jan. 14	Friday Jan. 15
Sky	☀	☁	⛅	☁	☁
Wind (mi/h)	5 North	4 East	11 North	0	11 South
High (°F)	10	6	−3	−4	0
Low (°F)	−2	−11	−16	0	−9

3. _____ yd² The screen in the movie theater measures 33 feet by 27 feet. In square yards, what is the area of the movie screen?

4. _____ A sequence is defined as shown for $n = 3, 4, 5, ...$ What will be the first number in this sequence that is NOT prime?

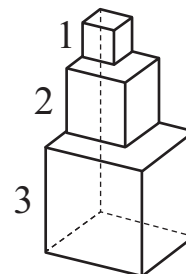$a_1 = 11$
$a_2 = 2$
$a_3 = a_1 + a_2$
$a_4 = a_3 + (a_2 + 2)$
$a_5 = a_4 + (a_2 + 4)$
$a_6 = a_5 + (a_2 + 6)$
$\vdots \qquad \vdots$
$a_{n+1} = a_n + (a_2 + 2(n - 2))$

5. _____ What is the smallest positive perfect cube divisible by 12?

6. _____ mi/h   While driving, Carl notices that his odometer reads 25,952 miles, which happens to be a palindrome. He thought this was pretty rare, but 2.5 hours later, his odometer reads as the next palindrome number of miles. What was Carl's average speed during those 2.5 hours?

7. _____   Let $a \blacklozenge b = \dfrac{ab}{a+b}$. What is the value of $(1 \blacklozenge 2) \blacklozenge 3$? Express your answer as a common fraction.

8. _____ %   Grapes are 80% water by weight. When a bushel of grapes dries in the sun for two weeks, it loses 50% of its total weight. All of the weight loss is due to the loss of water. After drying for two weeks, what percentage of the grapes is water by weight? Express your answer to the nearest percent.

9. _____   The number a is 5 times as large as b. The sum of a and b is 15. What is the value of the product ab? Express your answer as a common fraction.

10. _____ ordered pairs   Goldbach's conjecture states that every even number greater than two can be expressed as the sum of two prime numbers. For example, $2022 = 191 + 1831$. How many ordered pairs of prime numbers have a sum of 60?

11. _____ The function $f(x)$ has the property that if P and Q are any two distinct points on the graph of $y = f(x)$, then the slope of PQ is twice the sum of the x-coordinates of P and Q. What is the value of $f(10) - f(0)$?

12. _____ cm² The solid shown consists of three stacked cubes of edge lengths 1 cm, 2 cm and 3 cm. What is the surface area of the figure?

13. _____ If $|2x - 1| = |x - 2|$, what is the sum of all possible values of x?

14. _____ % Brian has earned 65%, 80% and 92% on his three pre-final exams. These exams are not weighed equally: the lowest counts for only 20% of his overall grade, while the other two count for 25% each. If the final exam is the remainder of the overall grade and there are no opportunities for extra credit, what is the highest grade Brian can earn in the class? Express your answer to the nearest whole percent.
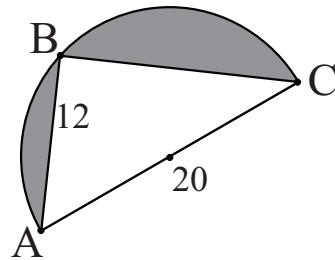
15. _____ units² A convex polygon has vertices at the points (2, 0), (1, 0), (1, 5) and (–2, 4). What is the area of the polygon?

16. _____ Let $x > 0$ and $y > 0$. Suppose that $xy^2 = 6$, and $x^2y^6 = 72$. What is the value of xy? Express your answer in simplest radical form.

17. _____ If $\sqrt{7 - \sqrt{2 + \sqrt{n}}} = 2$, what is the value of n?

18. _____ **integers** How many integers between 1 and 280, inclusive, are not divisible by 2, 5 or 7?

19. _____ In the figure, triangle ABC is inscribed in a semicircle with diameter AC of length 20 inches, and AB = 12 inches. When the area of the shaded region, in square inches, is expressed in the form $a\pi - b$, what is the value of $a + b$?

20. _____ If $n = 10^{2020} - 10^{2019} + 10^{2018} - 10^{2017} + \cdots + 10^2 - 10^1$, what is the sum of digits of the integer n?
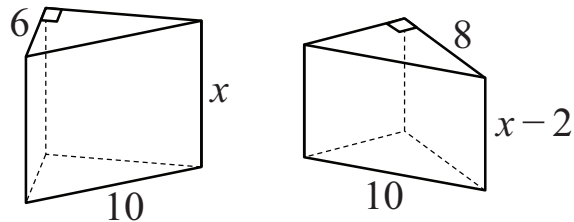
21. _____ cards Bertrand has 12 cards: on the front of each card is a digit, and on the back of each card is either a triangle, square, pentagon or circle. Bertrand looks through the pile of cards and says to his friend Georg, "For each card in the pile, if the number on the front of the card is even, then the shape on the back is a circle". Bertrand then lays the cards on a table as below. Georg wants to determine whether Bertrand was telling the truth by examining the hidden side of some of the cards. What is the fewest number of cards that Georg must examine to determine whether Bertrand was telling the truth?

22. _____ cm Both of the right triangular prisms shown have bases that are right triangles with hypotenuse length 10 cm. One prism has height x and a base leg of length 6 cm. The other prism has height $x - 2$ and a base leg of length 8 cm. The prism with height $x - 2$ has a volume that is 75% the volume of the prism of height x. What is the value of x?

23. _____ A positive integer divisor of 11! is chosen at random. What is the probability that this divisor is prime? Express your answer as a common fraction.

24. _____ ways Thomas, Carrie and Lenny each captain a different one of three hockey teams. Each captain will choose four players from a pool of 12 players, with each player chosen for only one team. How many different ways can the teams be formed?

25. _____ What is the greatest integer k such that 80! is divisible by 45^k?

26. _____ If $(7^2 + 24^2)^4 \times (5^2 + 10^2)^5 \times (75^2 + 100^2)^6 = 5^n$, what is the value of n?

27. _____ If m and n are positive integers where $m^2 + 14m - 32 = 3^n$, what is the value of $m + n$?

28. _____ The quadratic polynomial $p(x) = ax^2 + bx + c$ has positive integer coefficients a, b and c that each have a value from 1 to 9, inclusive. To eight decimal places, the solutions to $p(x) = 0$ are $x = -0.19098301$ and $x = -1.30901699$. What is the value of $p(10)$?

29. _____ If the expression $\dfrac{11^{20} + 9^{20}}{11^{20} - 9^{20}}$ is evaluated and written as a decimal, what digit is in the hundredths place?

30. _____ John inserts some number of parentheses into the expression shown to create a valid mathematical expression. What is the smallest possible integer value of John's expression?

$$1 \div 2 \div 3 \div 4 \div 5 \div 6 \div 7 \div 8 \div 9 \div 10$$