

# MATHCOUNTS® Problem of the Week Archive

## Falling in Fall – November 29, 2021

### Problems & Solutions

When thinking of things that “fall in the fall,” perhaps the first things that come to your mind are *leaves*? For many people around the country, the leaves are falling from their trees and hours of raking are necessary. Joe decided that he would try to make some extra money by raking the leaves in his neighbors’ yards. Joe figured he would need approximately seven bags per yard in which to collect the leaves. The bags are sold 10 to a box, and Joe is planning on raking 14 yards. How many boxes of bags must he buy?

*If about seven bags are needed for each yard, and Joe is raking 14 yards, he will need  $7 \times 14 = 98$  bags. The bags are sold in boxes of 10, so the 98 bags Joe will use are in  $98 \div 10 = 9.8$  boxes. Therefore, he must purchase **10** boxes of 10 bags to rake the 14 yards.*

Another thing that seems to “fall in the fall” is the *temperature*. Heather noticed that the high temperature on October 21st was 66 degrees, the high temperature on October 26th was 63 degrees and the high temperature on October 31st was 60 degrees. It appears that the high temperatures on every fifth day are forming an arithmetic sequence. If this sequence were to continue, on what date would the high temperature be 33 degrees?

*We could certainly list out the next temperatures (subtracting 3 degrees each time) with their respective dates (counting out five days each time), but let’s approach this from a different angle. We need the temperature to go down  $60 - 33 = 27$  more degrees after October 31. This is  $27 \div 3 = 9$  additional 3-degree decreases. This 9<sup>th</sup> decrease will take place on the  $9 \times 5 = 45^{\text{th}}$  day after October 31. November has 30 days, so the high temperature would be 33 degrees on **December 15<sup>th</sup>**.*

A third thing that tends to “fall in the fall” is people’s *amount of physical activity*. It is believed that people generally gain weight in the colder months in large part due to this lack of physical activity. Lou decided to be pro-active about this and promised himself he would exercise at least four days of every calendar week (or every Sunday through Saturday time period). If Lou never wants to skip exercising on consecutive days within a calendar week, and he exercises exactly four days out of the seven, how many different workout schedules are possible for a calendar week?

*The first thing to notice is that Lou can miss a Saturday and then the next Sunday. Those two days are in different calendar weeks. We’re only concerned with skipping consecutive days within a Sunday through Saturday period. Rather than listing the four days he could exercise, let’s list the three days that he can miss. (S = Sunday, M = Monday, T = Tuesday, W = Wednesday, R = Thursday, F = Friday and Q = Saturday.) If he misses exercising on the first Sunday, he has these possible “missed days” options: (S, T, R), (S, T, F), (S, T, Q), (S, W, F), (S, W, Q) and (S, R, Q). If he exercises on Sunday, and his first missed day is Monday, then he has these options: (M, W, F), (M, W, Q) and (M, R, Q). If he exercises on Sunday and Monday and his first missed day is Tuesday, then he has only the option of missing (T, R, Q). If he works out on Sunday, Monday and Tuesday, then there is no way to work out only one more day and not miss exercising on consecutive days. Therefore, there are only these **10** “missed days” options.*

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### ***Problems***

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