MATHCOUNTS® Problem of the Week Archive

2021 School Competition – November 8, 2021

Problems & Solutions

Now that the 2022 School Competition has been released, here are some problems from the 2021 equivalent to the School Competition for extra practice!

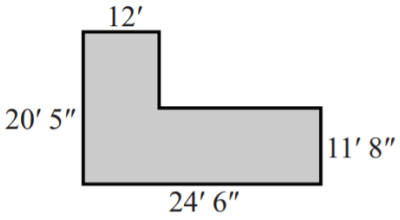
Sprint Round, #18

Jan and Jerome are mixing red paint with white paint to make pink paint, in the ratio of 4 parts red to 5 parts white. How many gallons of red paint should they mix with 1 gallon of white paint? Express your answer as a common fraction.

At a ratio of 4 parts red to 5 parts white, the amount of red in the pink is 4/5 the amount of white. To make this pink paint, the amount of red that should be mixed with 1 gallon of white is 1 × 4/5 = ***4/5*** *gallon.*

Sprint Round, #24

This figure shows the floorplan of a room that includes a door and a window that have a combined area of 50 ft^2. Each side of the room has a wall that is 8 feet tall, and adjacent walls meet at right angles. If a can of paint covers an area of 400 ft2, how many whole cans of paint must be purchased to paint the interior walls of this room, not including the door and the window?



The perimeter of the room is $2(24\ ^{6}/_{12} + 20\ ^{5}/_{12}) = 2 \times 44\ ^{11}/_{12} = 88\ ^{11}/_{6} = 89\ ^{5}/_{6}$ *ft. Not including the door and window, the walls have a total area of* $89\ ^{5}/_{6} \times 8 = 712\ ^{20}/_{3} = 718\ ^{2}/_{3}\ ft^2$*. Taking out the area of the window and door, the combined area of walls that need to be painted is* $718\ ^{2}/_{3} - 50 = 668\ ^{2}/_{3}\ ft^2$*. If a can of paint covers 400* ft^2*, a single can is not enough to cover the walls, but 2 cans of paint cover 400 × 2 = 800* ft^2*, which is more area than the* $668\ ^{2}/_{3}\ ft^2$ *that need to be painted. So,* ***2*** *whole cans need to be purchased.*

Target Round, #3

Jamie makes 120 slices of toast and puts at most one spread (jam, butter or avocado) on each slice. She puts jam on 1/3 of the slices, butter on 15 of the slices and avocado on 50% of the slices. How many slices of Jamie's toast have no spread?

Jamie puts jam on 1/3 × 120 = 40 slices. She puts butter on 15 slices. She puts avocado on 0.50 × 120 = 60 slices. That means the number of slices with no spreads is 120 – (40 + 15 + 60) = 120 – 115 = ***5*** *slices.*

Target Round, #6

There are 10 coins on a table heads side up. Noah wants them all to be tails side up, but with each move, he must turn over exactly 6 coins. What is the fewest moves he can take so that he ends up with all of the coins tails side up?

We know that Noah's last move must be flipping all 6 coins that are heads up to tails up so that he ends up with all 10 coins that are tails up. Noah starts with all 10 coins that are heads up [HEADS = 10, TAILS = 0]. In his first move, Noah must flip 6 coins from heads up to tails up [HEADS = 10 – 6 = 4, TAILS = 0 + 6 = 6]. There is no way for his second move to be flipping 6 coins that are heads up to tails up so that he ends up with all 10 coins that are tails up, but let's see if his second move can result in 6 coins that are heads up and 4 coins that are tails up. Let h represent the number of coins Noah flips from heads up to tails up, and let t represent the number of coins he flips tails up to heads up. For the 4 coins that are heads up, Noah needs 4 – h + t = 6, which simplifies to –h + t = 2. We also know that h + t = 6. Adding the equations –h + t = 2 and h + t = 6, we get 2t = 8, so t = 8/2 = 4, which also means that h + 4 = 6, and h = 6 – 4 = 2. So, in his second move, Noah needs to flip 2 coins from heads up to tails up and 4 coins from tails up to heads up [HEADS = 4 – 2 + 4 = 6, TAILS = 6 – 4 + 2 = 4]. Now, in his third move, Noah can flip the 6 coins that are heads up to tails up so that all 10 coins are tails up [HEADS = 6 – 6 = 0, TAILS = 4 + 6 = 10]. Therefore, Noah must make at least ***3*** *moves.*

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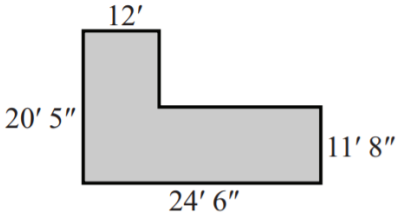

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