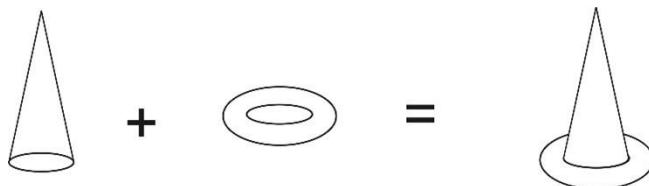


# MATHCOUNTS® Problem of the Week Archive

**Happy Halloween! – November 1, 2021**

## Problems & Solutions

Gretchen is going to make a witch's hat for Halloween. Her pattern consists of a right circular cone without a base attached to a circle with a hole cut out of the middle, as shown below. The hole is congruent to the base of the cone.



Gretchen plans to make the conical portion of her hat 18 inches tall with a base circumference of  $5\pi$  inches. What is the slant height of the conical portion of the witch hat if made according to Gretchen's plan? Express your answer as a decimal to the nearest tenth.

*Looking at the relationship between the vertical height, radius of the base and the slant height, we find that it creates a right triangle, so we can use the Pythagorean theorem to solve:  $(radius)^2 + (vertical height)^2 = (slant height)^2$ . So, let's start by finding the radius of the base using the given circumference of  $5\pi$  inches:  $C = 5\pi = 2\pi r$ . Solving for  $r$ , we find that  $r = 5\pi/2\pi = 5/2 = 2.5$  inches. Now, we can substitute what we know into the Pythagorean theorem:  $2.5^2 + 18^2 = (slant height)^2 \rightarrow 6.25 + 324 = (slant height)^2 \rightarrow (slant height)^2 = 330.25 \rightarrow \text{slant height} \approx 18.2$  inches, to the nearest tenth.*

Gretchen's completed hat looks great. Unfortunately, when she tries to put her hat on, she realizes it is too small! When she double checks the dimensions, she finds that they are exactly as she had planned so she must have measured incorrectly in the beginning. Not wanting to waste the great looking hat, she decides to use it to put candy in. How many cubic inches of candy will exactly fill the conical portion of the hat? Express your answer as a common fraction in terms of  $\pi$ .

*To find the volume of a cone, we use the equation  $V = (1/3)\pi r^2 h$ . Using the dimensions from the previous question, we can substitute known values and solve:  $V = (1/3)\pi(5/2)^2(18) = 75\pi/2$  cubic inches.*

If the brim of Gretchen's hat is a ring that extends 4 inches out from the base of the conical portion, what is the area of the brim in square inches? Express your answer in terms of  $\pi$ .

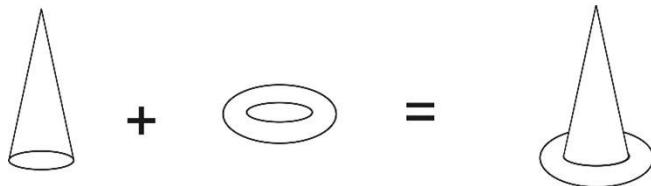
*Based on the first question, we know that the hole in the middle has a radius of 2.5 inches. Therefore, the overall radius of the outer edge of the brim is  $4 + 2.5 = 6.5$  inches. This means that if the brim were made of a solid piece of fabric, the area would be  $6.5^2\pi = 42.25\pi$  square inches. However, because the brim has a hole in it that matches the base of the conical portion of the hat, we need to subtract out the area of the middle hole. So,  $42.25\pi - 2.5^2\pi = 42.25\pi - 6.25\pi = 36\pi$  square inches.*

# MATHCOUNTS® Problem of the Week Archive

**Happy Halloween! – November 1, 2021**

## Problems

Gretchen is going to make a witch's hat for Halloween. Her pattern consists of a right circular cone without a base attached to a circle with a hole cut out of the middle, as shown below. The hole is congruent to the base of the cone.



Gretchen plans to make the conical portion of her hat 18 inches tall with a base circumference of  $5\pi$  inches. What is the slant height of the conical portion of the witch hat if made according to Gretchen's plan? Express your answer as a decimal to the nearest tenth.

Gretchen's completed hat looks great. Unfortunately, when she tries to put her hat on, she realizes it is too small! When she double checks the dimensions, she finds that they are exactly as she had planned so she must have measured incorrectly in the beginning. Not wanting to waste the great looking hat, she decides to use it to put candy in. How many cubic inches of candy will exactly fill the conical portion of the hat? Express your answer as a common fraction in terms of  $\pi$ .

If the brim of Gretchen's hat is a ring that extends 4 inches out from the base of the conical portion, what is the area of the brim in square inches? Express your answer in terms of  $\pi$ .