MATHCOUNTS® Problem of the Week Archive

Play Ball! – October 11, 2021

Problems & Solutions

The second round of the Major League Baseball play-offs is "best of 5," meaning that two teams will play each other until one team wins three games. How many different ways could this play-off match-up between Team A and Team B play out? (For example, here's 2 different ways the series could go with Team A winning: ABAA or AABA.)

*We can figure out how many ways Team A can win the play-off match-up and then double that number to account for the possibilities where Team B wins the match-up in the exact opposite ways. There are 3 general outcomes to consider: Team A could win 3-0, 3-1 or 3-2. First, if they win the match-up 3-0, there is only one way to do that. They would win the first three games (AAA). There are three possible ways for Team A to win the match-up 3-1**.** In this case, we know that there would only be four games, and one of the games won by Team A must be the fourth game (otherwise, Team A would have won the first three games, which is a scenario for which we have already accounted). Team B could win either the first, second or third game (BAAA, ABAA or AABA). Now, if the match-up ends 3-2, there are six ways this could play out. By the same logic, we know that Team A would have to win the fifth game, so we are really trying to figure out how many ways Team B could win two games out of the first four games: ${}_4C_2$ = (4!)/(2!2!) = 6 (BBAAA, BABAA, BAABA, ABBAA, ABABA or AABBA). So, there are 1 + 3 + 6 = 10 ways that Team A could win the match-up. If we were to switch all of the As for Bs and all of the Bs for As, we would find 10 ways that Team B could win in the same way. Therefore, there are **20** different ways a "best of 5" match-up could play out.*

There is a distance of 60 feet 6 inches between the batter and the pitcher, and an average fast ball travels 93 miles per hour. Assuming the speed of the ball remains constant, how many seconds will it take for the ball to travel 60 feet 6 inches? Express your answer to the nearest hundredth.

*We are given the speed of the ball in miles per hour and the distance the ball travels in feet/inches, so let's start by converting 93 miles per hour into feet per second. There are 5280 feet in one mile and 3600 seconds in an hour, so (93 miles/1 hour) × (5280 feet/1 mile) × (1 hour/3600 seconds) = 136.4 feet per second. Now, the distance we are working with is 60 feet 6 inches, which is equivalent to 60.5 feet. Substituting the known values into the distance = rate × time formula, we have 60.5 = 136.4 × t → t = 60.5/136.4 → t ≈ **0.44** seconds, to the nearest hundredth.*

After hitting the ball, the batter must run 90 feet to first base. The four bases (1st, 2nd, 3rd and home plate) are set up in a square. How many feet/inches (for example, 8' 2") are in the distance between home plate and 2nd base, to the nearest inch?

*Ultimately, we are trying to find the length of the diagonal of a square that is 90 feet by 90 feet. Notice that a line drawn from home plate to 2nd base is the hypotenuse of a right triangle with 90-foot legs. Using the Pythagorean theorem, we see that the length of this diagonal, x, is $90^2 + 90^2 = x^2$ → 8100 + 8100 = x^2 → 16,200 = x^2 → x = 127.2792206136 feet. Now, we'll need to find 0.2792206136 feet in inches, so multiplying by 12 gives 3 inches, to the nearest inch. Thus, the total distance from home plate to 2nd base is **127' 3"**.*

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