

MATHCOUNTS[®] Problem of the Week Archive

Bag of Bricks – September 20, 2021

Problems & Solutions

Kaleb has a bag containing red, black and white LEGO[®] bricks. If he has 12 more red bricks than white bricks and 15 fewer white bricks than black bricks, what is the positive difference between the number of red bricks and black bricks in Kaleb's bag?

*Let r , b and w represent the number of red, black and white bricks, respectively. We are told that $r = w + 12$ and $b = w + 15$. From these two equations, we can see that there are 3 more black bricks than red bricks. So, the positive difference between the number of red bricks and black bricks in Kaleb's bag is **3** bricks.*

Kaleb is thinking about sharing his bag of LEGO[®] bricks with some friends. Knowing the information given in the previous problem and that Kaleb has a total of 117 red, black and white bricks in his bag, what is the maximum number of friends with whom Kaleb can share his bricks so that all the bricks of each color are evenly distributed among Kaleb and his friends with no bricks leftover?

*We first need to determine the number of bricks Kaleb has of each color. We are told that $r + b + w = 117$. From the previous problem, we know that $r = w + 12$ and $b = w + 15$. We can substitute these two expressions for r and b in the first equation to get $(w + 12) + (w + 15) + w = 117$. Solving for w , we get $3w + 27 = 117 \rightarrow 3w = 90 \rightarrow w = 30$ bricks. So, there are 30 white bricks. That means there are $r = 30 + 12 = 42$ red bricks and $b = 30 + 15 = 45$ black bricks. We are looking for the largest number that is a factor of 30, 42 and 45. Finding the prime factorization of each of these numbers, we see that $30 = 2 \times 3 \times 5$, $42 = 2 \times 3 \times 7$ and $45 = 3 \times 3 \times 5$. The greatest common factor of 30, 42 and 45 is 3. Therefore, the bricks of each color can be divided into 3 groups evenly with no bricks leftover. That means the bricks can be divided among Kaleb and **2** friends.*

Instead of sharing with friends, Kaleb decides to share his bag of LEGO[®] bricks with his sister, Kami. Kaleb randomly selects one brick from the bag and gives it to Kami, then randomly selects another brick from the bag, without replacement, and keeps it for himself. Kaleb continues this random selection and distribution process until he and Kami have the same number of bricks and there is one brick remaining in the bag. If Kaleb started with all 117 LEGO[®] bricks in his bag, what is the probability that the brick left in the bag is black? Express your answer as a common fraction.

From the previous problem, we determined that Kaleb has 45 black bricks in his bag. Therefore, the probability that the brick left in the bag will be black is $45/117 = 5/13$.

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