MATHCOUNTS® Problem of the Week Archive

School Days – August 30, 2021

Problems & Solutions

Mrs. Marble refers to the three students named Ashley in her 3rd period gym class as Ashley G., Ashley E. and Ashley V. Mrs. Marble needs to divide the class to make 6 teams of 5 players for the dodge ball tournament. Each student has been assigned a number from 1 to 30. Mrs. Marble has placed 30 cards, each containing a number from 1 to 30, in a bowl. She will randomly select groups of 5 cards, without replacement to make the teams. What is the probability that all three Ashleys will be on the same team? Express your answer as a common fraction.

We just need any one of the groups of five to have all three Ashleys. Three of the students in the group we know, and the other two students could be any of their 27 classmates. There are $_{27}C_2 = (27 \times 26)/(2 \times 1) = 702/2 = 351$ *such combinations. But the girls could be selected for one of six teams. That brings the total to* $351 \times 6 = 2106$ *combinations. There are* $_{30}C_5 = (30 \times 29 \times 28 \times 27 \times 26)/(5 \times 4 \times 3 \times 2 \times 1) = 17{,}100{,}720/120 = 142{,}506$ *possible combinations of five students. So, the probability that one of the teams will include all three Ashleys is* $2106/142{,}506 =$ ***3/203****.*

On the first day of class, Willem's Algebra I teacher agreed to assign no homework for a week to the first student to answer this question: There is a positive two-digit integer that is divisible by the sum of its digits and has exactly five factors. What is the sum of its factors?

There are quite a few two-digit integers that are divisible by the sum of their digits, so let's first consider two-digit integers that have five factors. Since the number of factors is odd, we know this integer must be a perfect square. The only two-digit perfect squares are 16, 25, 36, 49, 64 and 81. We see that 16 is not divisible by 7; 25 is not divisible by 7; 36 is divisible by 9; 49 is not divisible by 13; 64 is not divisible by 10; and 81 is divisible by 9. The integer can't be 36 because it has nine factors, which leaves 81. This integer does have five factors, and the sum of the factors of 81 is $1 + 3 + 9 + 27 + 81 =$ ***121****.*

Mr. Jolie gives a test every Friday that the day of the month is either a perfect square or a prime number. If Mr. Jolie gives four tests in a month, what is the sum of the days of the month on which he gave a test?

The possible sets of days for Fridays in a month are {1, 8, 15, 22, 29}, {2, 9, 16, 23, 30}, {3, 10, 17, 24, 31}, {4, 11, 18, 25}, {5, 12, 19, 26}, {6, 13, 20, 27} and {7, 14, 21, 28}. For the first set of days, the month would have 2 tests. For the second set of days, there would be 4 tests. For the remaining sets, the number of tests would be 3, 3, 2, 1 and 1, respectively. There is only one set of days that results in 4 tests. The sum of the days of the tests is $2 + 9 + 16 + 23 =$ ***50****.*

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