

# MATHCOUNTS® Problem of the Week Archive

## Let's Use Logic – August 9, 2021

### Problems & Solutions

The following are answer choices to a multiple choice question. If only one of the answers is correct, which choice must be the correct answer?

1. All of the below
2. None of the below
3. All of the above
4. One of the above
5. None of the above
6. None of the above

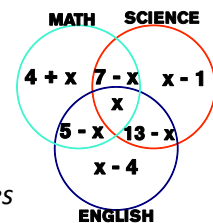
If choice 1 is true, then choice 2 must also be true, but the two are contradictory so we know neither can be true. This allows us to also eliminate 3, which states that both 1 and 2 are true. Next we can eliminate 4 which says that one of 1, 2 or 3 must be true, but we have established that none of these are correct choices. Choice 5 is true because it states that none of choices 1, 2, 3 and 4 are true. Lastly, choice 6 cannot be true since it implies that choice 5 is not true. Therefore, the only correct answer must be choice 5.

When Josh takes a survey of his 28 classmates, he gets the following results:

- 14 like English class
- 19 like Science class
- 16 like Math class
- 13 like English and Science class
- 7 like Science and Math class
- 5 like Math and English class

How many students like all English, Science and Math?

A Venn Diagram is a helpful tool in solving this problem. Because we are told 14 students like English, 19 like Science and 16 like Math, and  $14 + 19 + 16 = 49$ , which is greater than the number of students in the class, we know that some students have been counted multiple times. The number of students we are looking for will be in the intersection of all three circles (English, Science and Math) in the Venn Diagram. We can set this up and solve algebraically; we will label the intersection of all three circles in the diagram  $x$ , the quantity we are looking for, and write expressions for the other portions of the diagram. The value  $x$  (students who like all three subjects) is included in each of the values where two circles intersect (students who like two subjects), each of these intersections will have  $x$  subtracted from them to eliminate the double counting. For instance, where Math and Science intersect will be  $7 - x$  (the number of students that like both Math and Science minus the number of students who like Math, Science and English). The number of students who like only Math, only Science and only English are the numbers given 16, 19 and 14 respectively, minus the three intersections for each subject. For instance, the number of students who like only math will be  $16 - (7 - x) - (5 - x) - x = 4 + x$ . If we do



the same for the other subjects, the algebraic representations will appear as shown in the Venn Diagram. We can now solve for  $x$ . We know that the sum of all our expressions must equal 28, the total number of students in the class. We set up the equation and solve for  $x$ :

$$4 + x + 7 - x + 5 - x + x + x - 1 + 13 - x + x - 4 = 28 \rightarrow 24 + x = 28 \rightarrow x = 4$$

So, we find that the number of students that like all three subjects is 4.

If the following  $4 \times 4$  grid must be filled with letters A, B, C and D, so that each letter appears once in each row and once in each column, what is the letter that will appear in the square labeled  $\blacklozenge$ ?

B			$\blacklozenge$
	A		
		C	B
D			

There are many different places you can begin to solve this puzzle. Let's give each square a number in order to go through the steps of one possible solution. Starting in the upper left hand corner and moving to the right and down, we will number the squares 1 to 16 as shown. Starting at square 5, we can fill in C. We know this because there is an A in the same row and a B and D in the same column. This allows us to fill in square 9 with an A to complete the first column. Next, we can complete the third row by filling in square 10 with a D. Square 2 can be filled in with the letter C because there is a B in the same row and an A and D in the same column. Square 8 can be filled in with a D now because there is an A and C in the same row and a B in the same column. Now we have enough of the grid filled in to find the square labeled  $\blacklozenge$ . We do not need to complete the entire grid. There are a B and C in the same row and a D in the same column; therefore, the square labeled  $\blacklozenge$  must be filled in with an A.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

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