# MATHCOUNTS® Problem of the Week Archive

## ‘Tis the Season! – December 21, 2020

## *Problems & Solutions*

Ariel and Thomas are playing the Dreidel game. In their version of the game, players take turns spinning the dreidel and take various actions based on which symbol is face up when the dreidel stops spinning. The four actions the spinner can take are (1) put two tokens in the pot, (2) take half of the tokens from the pot, (3) take all of the tokens from the pot (and all other players each deposit one token into the pot), or (4) take no action and the next player spins. Before they begin, the total number of tokens is divided evenly between them. To start the game, Ariel and Thomas each deposit one of their tokens into the pot. Ariel spins the dreidel first and must place two of her tokens in the pot. For Thomas’ first spin he does nothing. In the second round, Ariel spins and must take half the tokens from the pot, while Thomas’ spin then results in his placing two tokens in the pot. During the third round, Ariel does nothing after her spin. When Thomas spins, he takes all the tokens from the pot which is equal to 2/5 of the total number of tokens. How many total tokens does Thomas have after taking these tokens?

*Let T be the total number of tokens that Ariel and Thomas are using for the game. Then, to begin round 1, the pot has 2 tokens and Ariel and Thomas each have T/2 – 1 tokens. At the end of the first round, Ariel places two of her tokens in the pot, so there are a total of 4 tokens in the pot, and Ariel now has T/2 – 1 – 2 = T/2 – 3 tokens. The total number of tokens Thomas has remains unchanged.*

*At the end of the second round, Ariel’s number of tokens increases by 2 since she must take half of the 4 tokens in the pot. Thomas has to place two tokens in the pot, reducing his number of tokens to T/2 – 1 – 2 = T/2 – 3. Since Ariel takes two tokens from the pot and Thomas puts in two tokens, there are still a total of 4 tokens in the pot.*

*At the end of the third round, Ariel’s number of tokens remains unchanged. Thomas takes all 4 tokens from the pot, which brings his number of tokens to T/2 – 3 + 4 = T/2 + 1. The four tokens in the pot are equivalent to 2/5 of the total number of tokens in play. Therefore, (2/5)T = 4. Multiplying both sides of the equation by 5/2 gives us T = 4(5/2) → T = 10. If the total number of tokens in play is 10, then at the end of the third round, Thomas has T/2 + 1 = 10/2 + 1 = 5 + 1 =* ***6*** *tokens.*

Every year on the first day of Christmas my true love gives me one gift, a partridge in a pear tree. On the second day of Christmas, my true love gives me a total of three gifts, two turtle doves and a partridge in a pear tree. This will go on until the twelfth day of Christmas, when my true love gives me a total of 78 gifts: 12 lords a-leaping, 11 ladies dancing, 10 pipers piping, 9 drummers drumming, 8 maids a-milking, 7 swans a-swimming, 6 geese a-laying, 5 golden rings, 4 calling birds, 3 French hens, 2 turtle doves, and a partridge in a pear tree. Today I’m at home “alone” with the gifts from my true love. Assuming each person, and each bird, has two legs, what day of Christmas is it if there are now 200 legs in my home (not including mine)?

*The first day of Christmas brings a gift with two legs.*

*On the second day, the two turtle doves and new partridge bring another 6 legs, so there are 2 + 6 = 8 legs.*

*On the third day, the new hens, turtle doves and partridge bring an additional 12 legs for a total of 8 + 12 = 20 legs.*

*The fourth day brings four calling birds and more hens, turtle doves and another partridge, with an additional 20 legs, for a grand total of 20 + 20 = 40 legs.*

*Every gift except the golden rings brings an additional pair of legs. That means that on the fifth day, the same number of legs are added as on the fourth day, for a total of 40 + 20 = 60 legs.*

*On the sixth day, six geese are added, plus more calling birds, hens, turtle doves and another partridge. This group adds another 32 legs, and the total becomes 60 + 32 = 92 legs.*

*On the seventh day, 7 swans, plus more geese, calling birds, hens, turtle doves and another partridge arrive, adding 46 more legs. That brings the total number of legs to 92 + 46 = 138 legs.*

*On the eighth day, 8 maids and more swans, geese, calling birds, hens, turtle doves and another partridge join the group, with an additional 62 legs. The total now is 138 + 62 = 200 legs.*

*Therefore, it must be the* ***eighth day*** *of Christmas.*

Each year Desmond and his family take part in the cultural celebration of Kwanzaa. The observance centers around the seven principles of Kwanzaa: Umoja, Kujichagulia, Ujima, Ujamaa, Nia, Kuumba and Imani. Suppose Desmond writes each of the five letters of the seventh principle, Imani (Swahili for Faith) on its own index card. If Desmond places the five cards in a bag and randomly chooses three cards, what is the probability that he chooses the three letters of the fifth principle, Nia (Swahili for Purpose)? Express your answer as a common fraction.

*The bag contains five cards, each with one of the letters I, M, A, N and i (let's use I and i to differentiate between the two I's in the bag). We start by finding out how many combinations of 3 letters are possible, and then determine how many of those contain the three letters in Nia. Since order is not important, there are* ${}_5C_3 = 5!/[(2!)(3!)] = (5 \times 4)/(2 \times 1) = 20/2 = 10$ *combinations of 3 letters that can be selected from the bag. One combination that works is N, I, A, and the other is N, i, A. Therefore, the probability of selecting the three letters in Nia is 2/10 =* ***1/5****.*

Based on the previous problem, what is the probability that he chooses the three letters in the correct order, N-i-a? Express your answer as a common fraction.

*In this case, order is important, and there are* ${}_5P_3 = 5!/2! = 5 \times 4 \times 3 = 60$ *arrangements of three letters chosen from the bag of five. Only two of the 60 selections work: N-I-A and N-i-A. Thus, the probability of selecting the three letters of Nia in the correct order is 2/60 =* ***1/30****.*

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Each year Desmond and his family take part in the cultural celebration of Kwanzaa. The observance centers around the seven principles of Kwanzaa: Umoja, Kujichagulia, Ujima, Ujamaa, Nia, Kuumba and Imani. Suppose Desmond writes each of the five letters of the seventh principle, Imani (Swahili for Faith) on its own index card. If Desmond places the five cards in a bag and randomly chooses three cards, what is the probability that he chooses the three letters of the fifth principle, Nia (Swahili for Purpose)? Express your answer as a common fraction.

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