MATHCOUNTS® Problem of the Week Archive

Merry-Go-Round – July 12, 2021

Problems & Solutions

A favorite ride in amusement parks has been the merry-go-round, also called a carousel. During the early Twentieth Century, the golden age of the carousel, there were about 6000 machines with finely crafted wooden figures. Today, there are about 160 that remain. The carousel is regaining its popularity, and some are being restored to their original condition. Most carousels have a circular platform with concentric rings of wooden figures. The carousel spins on its axis. The figures farthest from the center travel at a greater rate than the figures closest to the center.

Julie gets on a horse that is 18 feet from the center of the circular platform, and August gets on a horse that is 12 feet from the center of the circular platform. The carousel begins to rotate on its axis and reaches a constant running rate of 4.5 revolutions per minute. At that rate, what is Julie's speed? What is August's speed? Express your answer in feet per second rounded to the nearest tenth.

*Julie travels the circumference of a circle that is $C = 2 \times 18 \times \pi$ feet. The total distance that Julie travels in one minute is 4.5 revolutions $\times$ $C = 4.5 \times 2 \times 18 \times \pi = 162\pi$ feet. Divide the total distance by 60 seconds per minute to get that Julie's rate is **8.5** ft/sec, rounded to the nearest tenth. August travels the circumference of a circle that is $C = 2 \times 12 \times \pi$ feet. The total distance that August travels in one minute is 4.5 revolutions $\times$ $C = 4.5 \times 2 \times 12 \times \pi = 108\pi$ feet. Divide the total distance by 60 seconds per minute to get that August's rate is **5.7** ft/sec, rounded to the nearest tenth.*

The maximum height of the seat on Julie's horse is 55 inches while the minimum height is 27 inches above the circular platform. Assume the seat on Julie's horse starts at its maximum height and moves up and down at a constant rate while the carousel spins on its axis at a constant rate. It takes 14 seconds for the seat on Katie's horse to go from its maximum height to its minimum height and back to its maximum height. What is the height of the seat on Julie's horse exactly one minute later?

*The seat on Julie's horse is at its maximum height every 14 seconds. The seat on her horse will be at its maximum height at 14, 28, 42 and 56 seconds. The seat of the horse will be moving down for 60 – 56 = 4 seconds after the last maximum height is reached. It takes 7 seconds for the seat of the horse to go from its maximum height to its minimum height, so (4/7) × (55 – 27) = 16 inches. The height of the seat on Julie's horse is 55 – 16 = **39** inches one minute later.*

The seats on Katie's and Cory's horses both start at their maximum height at the same time and move up and down at a constant rate. It takes 14 seconds for the seat on Katie's horse to go from its maximum height to its minimum height and back to its maximum height. It takes 12 seconds for the seat on Cory's horse to go from its maximum height to its minimum height and back to its maximum height. What is the least number of seconds later that the seats on both horses are at their maximum height at the same time? How many times does the seat on Katie's horse reach its maximum height during the time that passes until the next time the seats on both horses reach their maximum height at the same time?

*The next time the seats of both horses are at their maximum height at the same time is the least common multiple of 12 and 14, which is 84. Thus, the least number of seconds needed for the seats of both horses to both be at their maximum height at the same time is **84** seconds. The seat on Katie's horse reaches its maximum height every 14 seconds. The period of time that passes from when the seats on both horses are at their maximum height at the same time until they are again at their maximum height at the same time is 84 seconds. So, 84 ÷ 14 seconds per maximum height = 6 times at maximum height. The seat on Katie's horse will be at its maximum height **6** times during the 84 seconds.*

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