

MATHCOUNTS® Problem of the Week Archive

High School Graduation – July 5, 2021

Problems & Solutions

This year, New High School's graduation ceremony was held at the Convention Center. The only seats available for guests of the graduates were in the five balcony sections. There were a total of 1504 seats in the balcony sections. After all guests were seated, one-fourth of the balcony seats remained empty. If only guests sat in the balcony seats, and no guests sat elsewhere, how many guests attended the graduation ceremony this year?

Since $1/4$ of the balcony seats were empty, it follows that all of the guests were seated in the other $3/4$ of the balcony seats. So, there were $(3/4) \times 1504 = \mathbf{1128}$ guests who attended the graduation ceremony.

Robin graduated from New High School this year. Her final grade point average earned her the 21st place in their graduating class. If Robin's final average was higher than the final averages of 93.75% of the students in her graduating class, how many students were in New High School's Class of 2021?

From the information provided, we know that if the graduates were to line up in descending order of their final average, Robin would hold the 21st place in line. It follows that the students from the 22nd place to the end of the line represent the 93.75% of all the graduates with a final average less than Robin's. That means that the first 21 students represent $100 - 93.75 = 6.25\%$ of the graduates. Therefore, the total number of graduates was $21 \div 0.0625 = \mathbf{336}$ students.

At the Convention Center, where New High School's graduation was held, the floor seating alternated between rows of n seats and rows of $n - 1$ seats. With the exception of the top two students, New High School's 2021 graduates sat in floor seats, completely filling 23 consecutive rows. Based on the previous problem, what was the total number of students seated in the 1st and 23rd rows?

From the previous problem, we know that excluding the top two students leaves $336 - 2 = 334$ students who sat in the floor seats. We are told that the students took up a total of 23 rows, and the number of seats in each row alternated. That means there were 12 rows containing n seats each and 11 rows containing $n - 1$ seats each. Since the 23 rows were completely filled with the 334 students, we can write the equation $12n + 11(n - 1) = 334$. Simplifying and solving for n , we have $12n + 11n - 11 = 334 \rightarrow 23n - 11 = 334 \rightarrow 23n = 345 \rightarrow n = 15$. Since the rows alternated between having 15 seats and 14 seats, it follows that the 1st and 23rd rows each contained 15 seats. Therefore, the total number of students seated in these two rows was $15 \times 2 = \mathbf{30}$ students.

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