MATHCOUNTS® Problem of the Week Archive

What Integer Am I? – June 21, 2021

Problems & Solutions

Many students will have started or will be starting summer breaks very soon. Here are some fun math problems to keep you "thinking math" even during these summer months! These problems are taken from previous MATHCOUNTS School Handbooks.

Choose a number. Triple that number. Add 200. Double the result. Subtract 100. Divide by 4. Subtract 150% of the original number. I am the result no matter what number you started with. What integer am I? (As a bonus… can you explain why I am always the result?)

*Let's start with a nice number like 10. Triple it to 30. Add 200 to get 230. Double this to get 460. Subtract 100 to get 360. Divide by 4 to get 90. Subtract 150% of 10 (in other words, subtract 15) to get 75. Would it be 75 if we start with 20? Let's try it out. Tripling 20 gives us 60. Adding 200 gives us 260, which becomes 520 when we double it. Subtracting 100 gives us 420, and dividing that by 4 gives us 105. Finally, subtracting 150% of 20 (so, subtracting 30) gives us 75. So, the mystery integer is **75**. Why does this work? Let's start with any number x. Tripling x gives us 3x. Adding 200 gives us 200 + 3x. Doubling this gives us 400 + 6x. Subtracting 100 gives us 300 + 6x. Then, we divide by 4 and get 75 + (3/2)x. Notice that subtracting 150% of x is the same as subtracting (3/2)x, and when we do this, we are left with 75. Our original number, x, is completely removed from the equation by the time we get to the end, so we will always be left with **75**.*

A rule allows you to replace an integer n with another integer ($a \times b$) provided $n = a + b$, where a and b are positive integers. For example, 7 can be expressed as 2 + 5 and can therefore be replaced with (2 × 5) = 10. This is only one of three possible replacement values for 7. I am the greatest integer you can obtain by starting with the number 5 and applying the rule four times consecutively. What integer am I?

*Each time we apply the rule, we will obtain the largest replacement value a × b when a and b are closest to each other. Applying the rule once, we get 5 = 2 + 3 and 2 × 3 = 6. Using this result and applying the rule a second time, we get 6 = 3 + 3 and 3 × 3 = 9. A third time yields 9 = 4 + 5 and 4 × 5 = 20. A fourth time yields 20 = 10 + 10 and 10 × 10 = 100. So, the mystery integer is **100**.*

The sum of my digits is 11. I am a value greater than 50 and less than 100. When I am divided by 2, the result is a prime integer. What integer am I?

*The first two sentences give us a lot of information. The number must be 56, 65, 74, 83 or 92. Dividing each of these by 2 gives us the new list of 28, 32.5, 37, 41.5 and 46. Only 37 is prime, so our mystery integer is **74**.*

I am a two-digit integer. My tens digit is odd, but my units digit is even. I am greater than 65. I am a multiple of 9, but I am not a multiple of 5. What integer am I?

To be a two-digit integer greater than 65 with an odd tens digit, the number must start with a 7 or a 9. If the units digit is even, it can be 0, 2, 4, 6 or 8, so that doesn't help narrow down our search too much yet. We know, though, that the number is a multiple of 9, which means that the digits must add to a multiple of 9. If the tens digit is 7, the units digit must be 2, which is 72. If the tens digit is 9, then the units digit must be 0 or 9, which is 90 or 99. But remember, we can only use an even units digit, so we're down to 72 or 90. The number is not a multiple of 5, so that eliminates 90, and the mystery integer is ***72****.*

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