MATHCOUNTS® Problem of the Week Archive

Time for Yearbooks! – May 17, 2021

Problems & Solutions

The end of the school year is when students across the country get excited to see the school's yearbook! In Talia's yearbook, the students' individual photos are printed in rows of four pictures. Each picture is 1.25 inches wide, and the width of the page is 8.5 inches. The four pictures in a row are spaced on the page such that the amount of empty space is the same between each pair of consecutive pictures and at both ends of the row of pictures. What is the number of inches between a pair of consecutive pictures in a row? Express your answer as a common fraction.

There are four pictures with a combined width of 1.25 × 4 = 5 inches. The page is 8.5 inches across, which leaves 8.5 – 5 = 3.5 inches of empty space. Notice that if S = empty space and P = picture, the arrangement is SPSPSPSPS. This shows there are five empty regions in one row of four pictures, and each of these five regions must all be the same size. Dividing 3.5 by 5, we see that there is .7 or ***7/10*** *of an inch between two adjacent pictures in a row.*

After finding her own individual picture in the yearbook, Talia flipped to the Clubs section to find the picture of the Math Club. The club had five girls and four boys in the picture. Talia remembered the photographer's directions when the shot was taken: "All girls are in the back row and the boys are in the front row. The tallest girl should go in the middle and the two shortest girls should be on the two ends of the back row. The two shortest boys should be on the two ends of the front row, too." Since none of the nine students were the same height, what is the total number of photo arrangements that would have satisfied the photographer's instructions?

Let's just consider the girls. The tallest girl goes in the middle of the back row. There is no option there. The two shortest girls are placed on the two ends, but we're not told which end the shortest girl must be on. So, between the two shortest girls, either one could have gone on the left side, giving 2 options for the left-most back position, but once that's filled, there's only 1 option for the right-most back position. The second and fourth positions can be filled in either order by the two remaining girls. So, there are 2 options for the second position, and the girl left over goes to the fourth position. In total, then, there are 2 × 2 × 1 × 1 = 4 ways they could be arranged. (Notice that once the first and second positions are filled, the other girls have no options where to stand – it has already been determined.) The same is really true for the boys in the front row. They just don't have a tallest boy in the middle, but as we saw above, that didn't impact the number of arrangements of the girls. There are then 2 × 2 × 1 × 1 = 4 ways the boys could be arranged. Each of the 4 girl-arrangements can be paired with any of the 4 boy-arrangements, for a total of 4 × 4 = ***16*** *possible arrangements.*

Talia and her best friend both had favorite numbers. They were 7 and 10. So, they decided that they would either sign or write a message on every page of each other's yearbook on every page that was a multiple of 7 or 10. The yearbook has 76 pages. On how many pages of Talia's yearbook did her friend leave a message or signature?

There are ten multiples of seven in the book (ranging from 7 × 1 = 7 to 7 × 10 = 70). There are seven multiples of 10 in the book (ranging from 10 × 1 = 10 to 10 × 7 = 70). Notice, though, that page 70 was counted as a multiple of 7 and as a multiple of 10. Therefore, there were 10 + 7 – 1 = ***16*** *pages on which Talia's friend left a message or signature.*

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