

MATHCOUNTS[®] Problem of the Week Archive

National Competition – May 3, 2021

Problems & Solutions

On Monday, May 10th, 224 of the nation's most talented middle-school math minds will be competing in the 2021 Raytheon Technologies MATHCOUNTS National Competition. Even though the National Competition will be virtual this year, the stakes are still high, and the problems will be tough. Here are a few problems national competitors solved in 2019.

Sprint #12

How many ways are there to distribute 11 identical tokens into 5 baskets, labeled A through E, so that each basket contains an odd number of tokens?

We can solve this problem by first determining the combinations for 5 odd numbers that have a sum of 11. They are $7 + 1 + 1 + 1 + 1$, $5 + 3 + 1 + 1 + 1$ and $3 + 3 + 3 + 1 + 1$. Now, we'll count the number of ways to arrange each of these groups of five quantities to find out how many ways each combination of tokens can be distributed among baskets A through E. The number of ways to distribute 7 tokens in one basket and 1 token in each of four other baskets is $5!/4! = 5$ ways. The number of ways to distribute 5 tokens in one basket, 3 tokens in another basket and 1 token in each of three baskets is $5!/3! = 20$ ways. Finally, the number of ways to distribute 3 tokens in each of three baskets and 1 token in each of two baskets is $5!/(3! \times 2!) = 10$ ways. That's a total of $5 + 20 + 10 = 35$ ways.

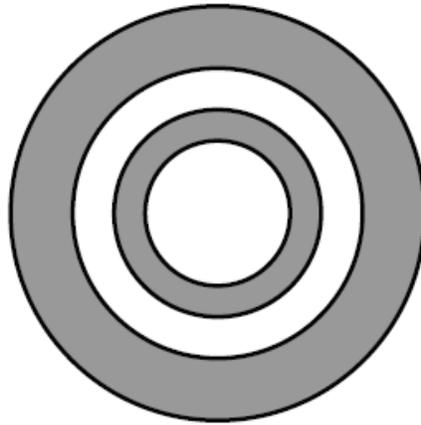
Sprint #17

Annie owns a floor sander that has two congruent rotating discs spinning at the same rate, with one disc in the front and the other in the rear. Both discs wear down at constant rates, but the front disc wears more per spin than the rear disc. A new disc placed in the front will completely wear out after 12,000 spins, while a new disc placed in the rear will completely wear out after 20,000 spins. Annie puts two new discs in this sander, which requires two operational discs to function. After how many spins should she switch the positions of the discs in order to be able to sand for the greatest amount of time with those two discs?

The front disc wears out after 12,000 spins, which is $3/5$ the number of spins it takes the rear disc to wear out. Likewise, the rear disc wears out after 20,000 spins, which is $5/3$ the number of spins it takes the front disc to wear out. We are trying to determine the number of spins s after which Annie should switch the positions of the discs to maximize the time with the two discs. She will get the maximum use out of the discs when they wear out at the same time. That occurs when $3/5 \times (20,000 - s) = 5/3 \times (12,000 - s)$. Simplifying, we get $12,000 - 3/5s = 20,000 - 5/3s$, or $180,000 - 9s = 300,000 - 25s$. Combining like terms and solving for s yields $16s = 120,000$, so $s = 7500$ spins.

Target #3

The four concentric circles shown have diameters 10 cm, 14 cm, 20 cm and 28 cm. What is the total area of the shaded regions? Express your answer to the nearest whole number.



Given the diameters of the concentric circles are 10 cm, 14 cm, 20 cm and 28 cm, it follows that they have radii 5 cm, 7 cm, 10 cm and 14 cm, respectively. The larger shaded region has an area equal to the difference between the areas of the largest and the second largest circles. The smaller shaded region has an area equal to the difference between the areas of the third largest and the smallest circles. Therefore, the total shaded area is $\pi[(14^2 - 10^2) + (7^2 - 5^2)] = 120\pi \approx \mathbf{377 \text{ cm}^2}$.

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