

# MATHCOUNTS<sup>®</sup> Problem of the Week Archive

## Tommy's X-treme Smoothies – April 26, 2021

### Problems & Solutions

At Smoothies by Tommy, the X-treme Berry smoothie is made by combining four different kinds of fruit, sweetener and ice. Tommy has already added 14.4 oz of fruit and 1.2 oz of sweetener to the blender. If the X-treme Berry smoothie contains only fruit, sweetener and ice, how many ounces of ice does Tommy need to add to the blender to make an X-treme Berry smoothie, which is 35% ice?

*Method 1: We know that  $14.4 + 1.2 = 15.6$  oz of the smoothie accounts for  $100 - 35 = 65\%$  of the mixture. If we let  $s$  represent the total number of ounces in an X-treme Berry smoothie, we can write  $0.65s = 15.6$ . Solving for  $s$ , we see that the total number of ounces in Tommy's smoothie is  $s = 15.6/0.65 = 24$  oz. Therefore, Tommy must add  $24 - 15.6 = 8.4$  oz of ice to the blender to make a smoothie that is 35% ice.*

*Method 2: Let  $n$  represent the number of ounces of ice Tommy needs to add to the blender. We know that the total number of ounces in the smoothie is  $14.4 + 1.2 + n = 15.6 + n$ . Since 35% of this must be ice, we can write  $0.35(15.6 + n) = n$ . Solving for  $n$ , we see that the number of ounces of ice Tommy needs to add to the blender is  $5.46 + 0.35n = n \rightarrow 5.46 = 0.65n \rightarrow n = 8.4$  oz.*

Using the exact same amount of fruit and ice used in the X-treme Berry smoothie, Tommy creates the X-treme Berry Lite smoothie by replacing the 1.2 oz of sweetener used in the X-treme Berry smoothie with an equal amount of zero-calorie sugar alternative. The 14.4-oz fruit mixture, which accounts for 100% of the calories in the X-treme Berry Lite smoothie, accounts for only 70% of the calories in the X-treme Berry smoothie. If there are 280 calories in the X-treme Berry Lite smoothie, how many calories are in 1 oz of the sweetener used to make the X-treme Berry smoothie?

*Let  $c$  be the total number of calories in an X-treme Berry smoothie. Since 280 calories is 70% of the X-treme Berry smoothie's calories, we have  $0.7c = 280 \rightarrow c = 400$  calories. Of these 400 calories, we know that  $400 - 280 = 120$  calories must come from the 1.2 oz of sweetener. Therefore, the number of calories in 1 oz of the sweetener used to make the X-treme Berry smoothie is  $120 \div 1.2 = 100$  calories.*

The X-treme Berry smoothie costs \$5.00. Customers who want a boost of energy can order the Super X-treme Berry smoothie. There is an additional charge of 75 cents to turn an X-treme Berry smoothie into a Super X-treme Berry smoothie by having a protein supplement added. Paula placed an order for 5 smoothies and paid \$27.25, not including tax. If Paula's order contained both X-treme Berry and Super X-treme Berry smoothies, what is the positive difference in the total amount she paid for the X-treme Berry smoothies and the total amount she paid for the Super X-treme Berry smoothies?

Let  $a$  represent the number of X-treme Berry smoothies Paula ordered at a cost of \$5.00 each. Let  $b$  represent the number of Super X-treme Berry smoothies Paula ordered at a cost of \$5.75 each. From the information provided, we can write the following two equations:  $a + b = 5$  and  $5a + 5.75b = 27.25$ . We can rewrite the first equation as  $a = 5 - b$  and substitute for  $a$  in the second equation. We have  $5(5 - b) + 5.75b = 27.25 \rightarrow 25 - 5b + 5.75b = 27.25 \rightarrow 25 + 0.75b = 27.25 \rightarrow 0.75b = 2.25 \rightarrow b = 3$ . It follows that  $a = 5 - 3 = 2$ . So, Paula paid a total of  $5 \times 2 = \$10.00$  for the X-treme Berry smoothies she ordered, and she paid a total of  $27.25 - 10.00 = \$17.25$  for the Super X-treme Berry smoothies she ordered. That's a difference of  $17.25 - 10.00 = \$7.25$ .

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