MATHCOUNTS® Problem of the Week Archive

Prime Time – April 12, 2021

Problems & Solutions

In the 17th century, Christian Goldbach developed an idea about prime numbers. He stated that every even integer greater than 2 can be written as the sum of two prime numbers. He had strong evidence to support his idea, but he was unable to prove that his idea is true. Today, this idea is known as Goldbach's Conjecture and still remains to be proven even though there have been many attempts to do so.

What is the greatest positive difference between two prime numbers whose sum is 18?

The sum of 5 and 13 is 18, and the sum of 7 and 11 is 18. The greatest positive difference is 13 – 5 = ***8****.*

Which two-digit even whole number can be expressed as the sum of two prime numbers whose positive difference is the greatest?

Let's try out a few two-digit sums:

98 = 19 + 79 (The positive difference here is 79 – 19 = 60.)
96 = 7 + 89 (The positive difference here is 89 – 7 = 82.)
94 = 5 + 89 (The positive difference here is 89 – 5 = 84.)
92 = 3 + 89 (The positive difference here is 89 – 3 = 86.)
90 = 7 + 83 (The positive difference here is 83 – 7 = 77.)

No number less than 90 can be expressed as the sum of two prime numbers whose positive difference is greater than 86. Therefore, ***92*** *is the two-digit even whole number that can be expressed as the sum of two prime numbers whose positive difference is the greatest.*

Another conjecture, called the "Weak Conjecture," also stated by Goldbach, says that any odd integer greater than 5 can be expressed as the sum of 3 prime numbers. In how many ways can 21 be expressed as the sum of 3 prime numbers?

Below are the ways 21 can be expressed as the sum of 3 prime numbers:

21 = 3 + 5 + 13
21 = 3 + 7 + 11
21 = 5 + 5 + 11
21 = 7 + 7 + 7
21 = 2 + 2 + 17

Thus, there are ***5*** *different ways 21 can be expressed as the sum of 3 prime numbers.*

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