

# MATHCOUNTS<sup>®</sup> Problem of the Week Archive

## State Competition – March 22, 2021

### Problems & Solutions

The State Competition is happening later this week! Are you ready to compete? Let's try a few 2020 State Competition problems to get ready.

#### 2020 State Sprint Round, #9

The square of 207 can be computed by multiplying 200 by  $200 + x$  and adding 49. What is the value of  $x$ ?

We can set up and solve the following equation:

$$200(200 + x) = 207^2 - 49 = 207^2 - 7^2 = (207 + 7)(207 - 7) = (214)200 = 200(200 + 14)$$

So,  $x = 14$ .

#### 2020 State Sprint Round, #14

A checker starts at square 4 of the checkerboard shown here. At any time, it can move to any diagonally adjacent square below its current position. How many possible ways are there for the checker to move from square 4 to square 32?

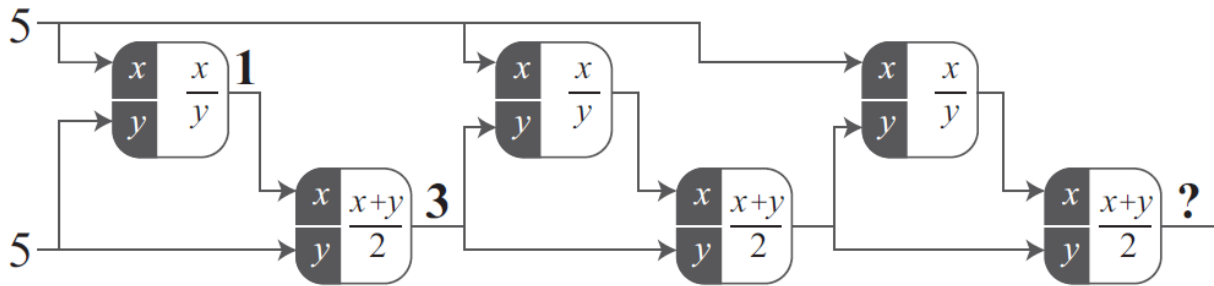
	1	2	3	4
5		6	7	8
	9	10	11	12
13		14	15	16
	17	18	19	20
21		22	23	24
	25	26	27	28
29		30	31	32

To get from square 4 to square 32 using only downward motions, one must pass through exactly one of squares 18, 19 and 20. For each of those 3 options, we can determine how many ways lead from square 4 to the square in question, and then how many ways lead from the square in question to square 32; since the ways from square 4 to the intermediate square are independent of the ways from the intermediate square to square 32, the number of ways from square 4 through the intermediate square under assessment to square 32 is the product of those two values. The total count of ways is the sum of those products over the 3 intermediate squares being examined. For square 18, there is 1 way from square 4 (4..8..11..15..18) and 1 way to square 32 (18..23..27..32). For square 19, there are 3 ways from square 4 (4..8..11..15..19; 4..8..11..16..19; 4..8..12..16..19) and 3 ways to square 32 (19..23..27..32; 19..24..27..32; 19..24..28..32). For square 20, there are 2 ways from square 4 (4..8..11..16..20; 4..8..12..16..20) and 2 ways to square 32 (20..24..27..32; 20..24..28..32). Therefore, the total number of qualifying ways from square 4 to square 32 is  $1 \times 1 + 3 \times 3 + 2 \times 2 = 14$  ways.

#### 2020 State Target Round, #2

Six function machines are connected as shown. Three of the function machines are “dividers”, while three of the function machines are “averagers”. Each divider takes two inputs (into the left) and sends their quotient to the output (on the right). Each averager takes two inputs (into the left) and sends their arithmetic mean to the output (on the right). Given that the initial inputs are  $x = 5$  and  $y = 5$ , as shown, what is the final result emitted by the right-most function machine? Express your answer as a mixed

number.



We have already been given that the output of machine 2 is 3, so we pick up from there. For machine 3, the  $x$  input is from the original upper input, 5, and the  $y$  input is the output of machine 2, so 3, and we are to divide the  $x$  value by the  $y$  value to result in an output of  $5/3$ . For machine 4, the  $x$  input is the output of machine 3, so  $5/3$ , and the  $y$  input is the output of machine 2, so 3, and we are to add the two inputs and then divide by 2 to result in an output of  $7/3$ . For machine 5, the  $x$  input is from the original upper input, 5, and the  $y$  input is the output of machine 4, so  $7/3$ , and we are to divide the  $x$  value by the  $y$  value to result in an output of  $15/7$ . For machine 6, the  $x$  input is the output of machine 5, so  $15/7$ , and the  $y$  input is the output of machine 4, so  $7/3$ , and we are to add the two inputs and then divide by 2 to result in an output of  $47/21$ , which is equal to the mixed number  **$2\ 5/21$** .

2020 State Target Round, #5

A remote island is populated by both truth-tellers and liars. Truth-tellers always tell the truth, and liars always lie. To conduct a census of the island, Erin gathers all 72 of its inhabitants and asks each person how many truth-tellers and liars there are. She receives 61 responses saying “there are 32 truth-tellers and 40 liars” and 11 responses saying “there are 11 truth-tellers and 61 liars”. How many liars are there?

The truth-tellers will correctly state how many truth-tellers there are, whereas the liars will state a wrong count of truth-tellers. The group of truth-tellers will be that group whose answer regarding the number of truth-tellers matches the number of people in the group. The only such match is 11, so the group claiming 11 truth-tellers are truth-tellers and they told the truth when they said there are **61** liars.

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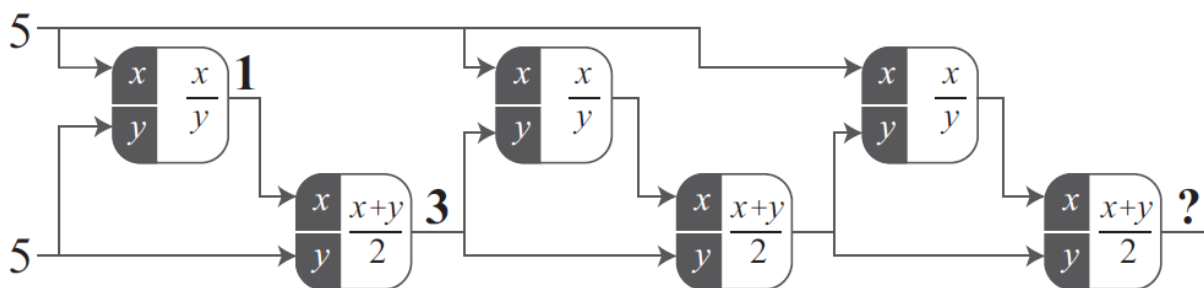
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