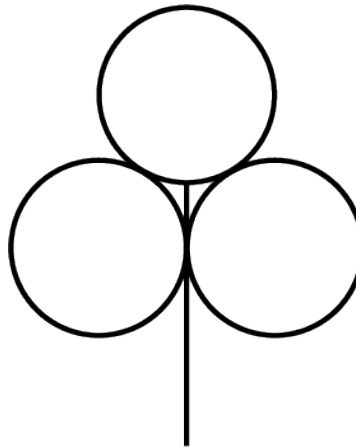


MATHCOUNTS[®] Problem of the Week Archive

Shamrock – March 15, 2021

Problems & Solutions

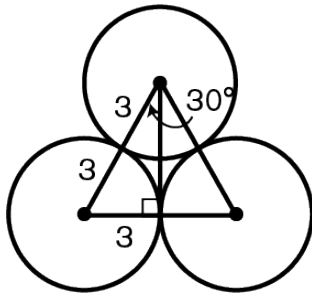
Below is a picture of the shamrock Sharon drew for St. Patrick's Day.



It consists of 3 congruent circles, each with a radius of 3 inches, that are tangent and a stem (line) that runs from the bottom of the upper circle to 6 inches below the point where the lower two circles touch.

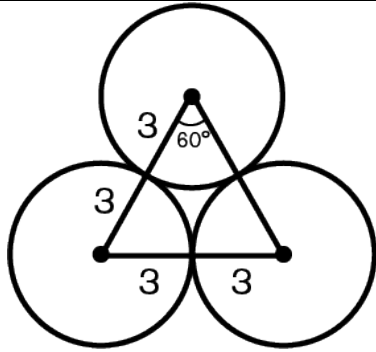
What is the length of the portion of the stem (line) that runs from the bottom of the upper circle to the point of tangency between the lower two circles? Express your answer in simplest radical form.

Notice that the three centers of the circles create an equilateral triangle.



Thus, the altitude of the triangle is $3\sqrt{3}$ inches, and the portion of the altitude that is not within the upper circle is $3\sqrt{3} - 3$ inches.

What is the length of the outer perimeter of the shape formed by the leaves of the shamrock? Express your answer in terms of π .



Notice that the perimeter of the shape is also the sum of the lengths of $300/360 = 5/6$ of the circumference of each of the circles. So, $C = 2\pi r = 2\pi(3) = 6\pi$ inches. Then, $(5/6)(6\pi) = 5\pi$ inches. Thus, the perimeter of the shape is $3(5\pi) = \mathbf{15\pi}$ inches.

What is the area of the space between the three leaves? Express your answer in simplest radical form, in terms of π .

We can solve this one by finding the area of the equilateral triangle discussed in the solution to problem #1 and then subtracting the areas of the three sectors of the circles that are within the triangle. The area of the triangle is $(1/2)(6)(3\sqrt{3}) = 9\sqrt{3} \text{ in}^2$.

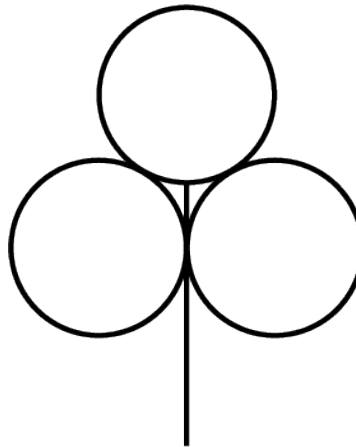
Since we know the vertex angle of each circle's sector contained in the triangle is 60 degrees, the area of each of the sectors is $(60/360)(\pi)(3^2) = (1/6)(\pi)(9) = 3\pi/2 \text{ in}^2$. There are three sectors of this same size within the triangle, so the area of the space between the three circles is $9\sqrt{3} - 3(3\pi/2) = 9\sqrt{3} - 9\pi/2$, or $\mathbf{(18\sqrt{3} - 9\pi)/2 \text{ in}^2}$.

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What is the length of the outer perimeter of the shape formed by the leaves of the shamrock? Express your answer in terms of π .

What is the area of the space between the three leaves? Express your answer in simplest radical form, in terms of π .