MATHCOUNTS® Problem of the Week Archive

2020 Chapter Competition – February 1, 2021

Problems & Solutions

With the 2021 Chapter Competition happening in a few days, here are some problems from the 2020 Chapter Competition for extra practice!

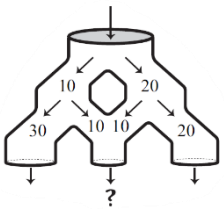
Sprint Round, #13

Rafa and Sascha played a long 320-point tennis match. If Rafa ran an average of 12.7 meters per point and Sascha ran an average of 11.8 meters per point, how many more meters did Rafa run over the course of the match?

Because Rafa averages moving 12.7 meters per point and Sascha averages 11.8 meters per point, Rafa averages 12.7 – 11.8 = 0.9 meters per point more than Sascha. Given 320 points for the match at that rate, Rafa will run a total of 320 × 0.9 = ***288*** *meters more than Sascha.*

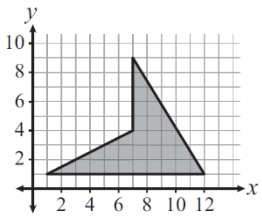
Sprint Round, #25

Ten thousand marbles are released into the top pipe as shown and roll down the pipe system. Anytime the pipe forks, the marbles split in proportion to the cross-sectional areas of the pipes. All pipes have circular cross-sections with diameters as indicated in the figure. How many marbles exit through the bottom, middle pipe?



The number of marbles that take one path versus the other at a fork is proportional to the ratio of the cross-sectional areas of the pipes for each path, which, in turn, is proportional to the squares of the given diameters. At the top fork, the left to right ratio of marbles is 10^2*:*20^2 *= 100:400 = 1:4. As fractions, the left path gets 1/(1 + 4) = 1/5 of the marbles and the right path gets 4/(1 + 4) = 4/5 of the marbles. Since there are 10,000 marbles, 10,000/4 = 2000 go left and the remaining 8000 go right at the top fork. The bottom row has three pipes: one on the left (L), one in the middle (M), and one on the right (R). The only one we really care about is M. The fork between L and M gets 2000 marbles, which are split proportionally* 30^2 *= 900 for L to* 10^2 *= 100 for M. Thus, 100/(900 + 100) = 1/10 of the 2000, or 200, go through M. The fork between M and R gets 8000 marbles, which are split proportionally* 10^2 *= 100 for M to* 20^2 *= 400 for R. Thus, 100/(100 + 400) = 1/5 of the 8000, or 1600, go through M. Therefore, the total going through M is 200 + 1600 =* ***1800*** *marbles.*

Target Round, #2



In the coordinate grid shown, what is the area of the shaded quadrilateral, all of whose vertices are on the lattice points of the grid?

*A vertical line along x = 7 splits the figure in question into two triangles: to the left of the line is a triangle of base 6 units and height 3 units for an area of 9 units^2, and to the right of the line is a triangle of base 5 units and height 8 units for an area of 20 units^2. The two triangles combined have area 9 + 20 = **29** units^2.*

Target Round, #3

If the polynomial $5x - 3$ is a factor of the polynomial $5x^2 + 7x + k$, what is the value of the constant k?

*The factor theorem says polynomial P(x) has x – r as a factor if and only if P(r) = 0, so 5x – 3 = 5(x – 3/5) is a factor if the polynomial has a value of 0 at x = 3/5: $0 = 5(3/5)^2 + 7(3/5) + k = 9/5 + 21/5 + k = 6 + k$, so k = **-6**. Alternatively, since 5x – 3 is a factor of the polynomial, then $(5x - 3)(x + a) = 5x^2 + 7x + k$. Expanding the left side of the equation yields $5x^2 + 5ax - 3x - 3a = 5x^2 + (5a - 3)x - 3a$. That must mean that 5a – 3 = 7 and 5a = 10, so a = 2. That also means that k = -3a = -3(2) = **-6**.*

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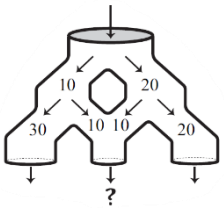
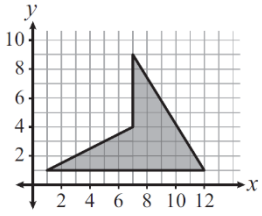


Target Round, #2



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