

# MATHCOUNTS® Problem of the Week Archive

## Hot Shot! – January 11, 2021

### Problems & Solutions

Hot Shot is an arcade basketball game in which the object is to make as many baskets as possible within the 60-second time limit. A player is awarded 2 points for each successful shot during the first 50 seconds of play and 3 points per successful shot during the final 10 seconds. Tyler had a bit of beginner's luck in her first game scoring a total of 60 points. She wants to exceed this total in her second game. What is the minimum number of baskets Tyler must make to earn a score greater than 60 points?

*To exceed her previous score, Tyler must score at least 61 points in her second game. The total number of baskets is minimized by maximizing the number of three-point baskets made. Let's start by determining the fewest number of two-point shots Tyler could make and earn a total of 61 points. If Tyler makes just one two-point shot, she would need to earn the remaining  $61 - 2 = 59$  points by making three-point baskets. But, since 59 is not divisible by 3, we know that it is not possible for her to earn the remaining points with three-point baskets. So, Tyler must make at least 2 baskets worth two points each, accounting for 4 points. In this case, she would need to earn an additional  $61 - 4 = 57$  points by making three-point baskets. Tyler could accomplish this by making  $57 \div 3 = 19$  three-point baskets. Therefore, Tyler must make a minimum of  $2 + 19 = 21$  baskets to exceed 60 points in her second game.*

Mitchell earned a total score of 55 points. Twenty percent of the baskets he made were worth three-points each. If four-fifths of the two-point baskets Mitchell attempted were successful and he made one-third of the three-point baskets he attempted, what is the total number of baskets that Mitchell attempted?

*Let  $X$  represent the number of three-point baskets Mitchell made. We are told that 20% or  $1/5$  of the baskets Mitchell made were worth three points. That means that  $4/5$  of the baskets he made were worth two points each, which is 4 times the number of three-point baskets. Therefore, the number of two-point baskets Mitchell made would be represented by  $4X$ . We are told that Mitchell's total score was 55 points. Thus,  $3X + 2(4X) = 55$ . Simplifying, we have  $3X + 8X = 55 \rightarrow 11X = 55$ . Dividing each side by 11 yields  $X = 5$ . So, Mitchell made 5 three-point baskets and  $4(5) = 20$  two-point baskets. Next, we are told that the 20 two-point baskets Mitchell made account for  $4/5$  of the total number of two-point baskets attempted. Therefore, Mitchell attempted  $(5/4) \times 20 = 25$  two-point shots. In addition, the 5 three-point baskets he made account for  $1/3$  of the three-point baskets attempted. Therefore, Mitchell attempted  $(3/1) \times 5 = 15$  three-point shots. Thus, Mitchell attempted to make a total of  $25 + 15 = 40$  baskets.*

Kyoka holds the world record for earning the most points in this arcade game with a high score of 256 points. If Kyoka did not miss any of the 114 shots she took to set this record, what percent of those shots were made during the final 10 seconds of the game? Express your answer to the nearest percent.

*The shots made during the final 10 seconds of the game would have been worth three points each, so we need to determine how many of the 114 shots were three-point baskets. We are told that Kyoka's total score was 256 points. So, if we let  $X$  represent the number of two-point baskets and  $Y$  be the number of three-point baskets, we have the following two equations:  $X + Y = 114$  and  $2X + 3Y = 256$ . Solving the first*

equation for  $X$  yields  $X = 114 - Y$ . We can substitute this expression in for  $X$  in the second equation to get  $2(114 - Y) + 3Y = 256$ . Simplifying, we have  $228 - 2Y + 3Y = 256 \rightarrow 228 + Y = 256$ . After subtracting 228 from each side, we have  $Y = 28$ . Thus, there were 28 shots made during the final 10 seconds of the game. That accounts for  $28/114 = 0.245614 \approx \mathbf{25\%}$  of the shots.

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### ***Problems***

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