FUTURE ENGINEERS

BAE SYSTEMS

MATHCOUNTS®

#EWEEK2021

FEB. 22–26 • SOLUTIONS TO E-WEEK PROBLEM SETS

MONDAY, FEB. 22: ENVIRONMENTAL ENGINEERING

1.1 Since the maximum allowed micrograms of contaminant *c* yields the maximum 5 ppb for safe groundwater, we can set up the following equation and solve for *c*:

$$5 = \left(\frac{c}{7 \times 10^8}\right) \times 10^9 = \frac{c \times 10^9}{7 \times 10^8} = \frac{10c}{7}$$

Therefore, $35 = 10c$ and $c = 3.5$. Thus, there could be a maximum of **3.5 micrograms** of contaminant in the groundwater for it to still be considered safe.

1.2 The volume of a square pyramid is calculated using the formula $V = a^2(h/3)$, where a = the length of a base edge and h = the height of the pyramid. So, the volume of this plume is $30^2(50/3) = (900 \times 50)/3 =$ **15,000 m³**.

1.3 Since the depth of the plume triples each year after the first year, the plume would be 50 × 3 = 150 m deep after the second year, 150 × 3 = 450 m deep after the third year, and 450 × 3 = 1350 m deep after the fourth year. Since the base edge of the plume doubles each year after the first year, the plume would be 30 × 2 = 60 m wide after the second year, 60 × 2 = 120 m wide after the third year, and 120 × 2 = 240 m wide after the fourth year. Therefore, the volume of the plume exactly 4 years after the solvent was dumped is $240^2(1350/3) = 57{,}600 \times 450$ = **25,920,000 m³**.

1.4 From 1.3, we know that the base edge of the plume after 4 years is 240 m. This means that above ground, an area of 240 m by 240 m, or 240 × 240 = **57,600 m²** must be blocked off to cover the entire plume.

TUESDAY, FEB. 23: SYSTEMS ENGINEERING

2.1 We can order the 11 coefficients of drag from the table to find the median C_d of a typical sports car:

0.27, 0.27, 0.28, 0.292, 0.31, 0.314, 0.33, 0.341, 0.35, 0.363, 0.377

The median value is the middle value in this list, which is **0.314**.

2.2 Frontal area is calculated as 85% of height × width, so the frontal area of Sedan A is 0.85 × 1.48 × 1.78 ≈ **2.239**.

2.3 Using the C_d from 2.1 and the frontal area of Sedan A calculated in 2.2, the C_dA of Sedan A is 0.314 × 2.239 ≈ 0.703. The C_dA of Sedan B is 0.314 × (0.85 × 1.41 × 1.85) ≈ 0.696. Finally, the C_dA of Sedan C is 0.314 × (0.85 × 1.37 × 1.91) ≈ 0.698. Therefore, Kami should choose **Sedan B**.

WEDNESDAY, FEB. 24: SOFTWARE ENGINEERING

3.1 The point in the second quadrant has coordinates (–3, 4), and the point located on the *x*-axis has coordinates (3, 0). First, we'll need to use the slope formula to determine the slope of the line that will pass through these two points. So, $m = (y_2 - y_1)/(x_2 - x_1) = (0 - 4)/(3 - (-3)) = (-4)/6 = -2/3$. Next, we can use the slope-intercept equation of a line $y = mx + b$ to determine the *y*-intercept *b*. So, we have slope $m = -2/3$ and we'll use the point (3, 0). Substituting for *x*, *y* and *m* gives us $0 = (-2/3)(3) + b$. Now, solving for *b*, we get $0 = -2 + b$ and $2 = b$. Thus, the equation in slope-intercept form that can be used to create a line that passes through these two points is $\mathbf{y = (-2/3)x + 2}$.

3.2 Notice that there is a constant rate of change in the base price and in the total cost. This tells us that the algorithm will be a linear equation, and we can write the algorithm as an equation in slope-intercept form, $y = mx + b$, where *y* = total cost and *x* = base price. So, let's use two data points from the table, (25, 32.50) and (50, 60), to determine the rate of change of this linear relationship: (60 – 32.50)/(50 – 25) = 27.50/25 = 1.1. Now, using this rate of change and a data point from the table, we can substitute and solve for *b*, the flat shipping rate: $y = mx + b \rightarrow 60 = 1.1(50) + b \rightarrow 60 = 55 + b \rightarrow 5 = b$. Thus, the algorithm used by this website is $\mathbf{y = 1.1x + 5}$.

3.3 Checksum does not catch errors in messages that have the correct three letters in the incorrect order. There are 3 × 2 × 1 = 6 arrangements of the letters BAD, of which only 1 is the correct message, so there are 6 – 1 = 5 ways that the message could have the correct three letters with an error that checksum does not catch. The sum of B + A + D = 2 + 1 + 4 = 7, so we must also consider any three-letter arrangements with incorrect letters that also sum to 7. For example, E + A + A = 5 + 1 + 1 = 7 as well, but this is obviously the wrong message. In total, there are 9 ways that checksum could miss an error with incorrect letters involved (EAA, AEA, AAE, BBC, BCB, CBB, ACC, CAC, CCA), as the sum of each of these arrangements is also 7. So, checksum would miss a total of 5 + 9 = 14 errors. There is a total of 26 × 26 × 26 = 17,576 possible three-letter arrangements, so the probability that you receive your friend's message with an error that checksum does not catch is 14/17,576 = **7/8788**.

THURSDAY, FEB. 25: AEROSPACE ENGINEERING

4.1 We'll need to use the rocket equation to determine the mass ratio. First, however, we need to calculate the *change in velocity*, which is (the change in the rocket's position) ÷ (the change in time). So, *change in velocity* = 940,000/158 = 470,000/79 m/s. Now, we can use this to evaluate the rocket equation:

$$\frac{m_{initial}}{m_{final}} = 2.718^{(470{,}000/79)/2000} \approx \mathbf{19.6}.$$

4.2 We now know that the mass ratio is equal to 19.6. So, $19.6 = m_{initial}/m_{final}$. We are told that the final mass m_{final} = 41,255 kg, so we have $19.6 = m_{initial}/41{,}255$. Multiplying both sides of the equation by 41,255 gives $m_{initial}$ = 808,598 kg. Thus, the fuel would make up 0.91 × 808,598 = **735,824.18 kg** of the initial mass.

4.3 For this problem, we'll need to use the formula *distance* = *rate* × *time*. We are told that the satellite travels a distance of 43,200 km at a rate of 8 km/s, so we have 43,200 = 8 × *time* → *time* = 43,200/8 = 5400 seconds. Note, however, that we are asked for the number of minutes for the satellite to complete one orbit. So, one orbit takes 5400/60 = **90 minutes**.

4.4 To determine how far above the earth's surface the satellite orbits, we'll need to find the difference between the radius of the satellite's orbit r_1 and the radius of the earth r_2. We can use the formula *circumference* = $2\pi r$ to determine r_1 since we know from the previous problem that the circumference of the satellite's orbit is 43,200 km. We have $43{,}200 = 2\pi r_1 \rightarrow 43{,}200/(2\pi) = r_1$. We can use the same formula to determine r_2 since we know the earth's circumference is 40,075 km. We have $40{,}075 = 2\pi r_2 \rightarrow 40{,}075/(2\pi) = r_2$. Therefore, the satellite orbits $r_1 - r_2 = 43{,}200/(2\pi) - 40{,}075/(2\pi) \approx$ **497.4 km** above the earth's surface.

FRIDAY, FEB. 26: CHEMICAL ENGINEERING

5.1 A total of 12 + 0.5 + 0.8 = 13.3 liters does not make it to the filtered solution. This is a decrease of (13.3/700) × 100 = 1.9%. So, the volume of the solution decreases by approximately **2%**.

5.2 From the point where the solution initially enters the piping to the filter, there are 5 + (18 – 6) = 17 feet. The solution travels 6 inches per second during this time. So, we can perform the following calculation and conversion: 17 ft × (1 sec/6 in) × (12 in/1 ft). Canceling the 6 and 12 leaves 2 × 17 = 34 seconds. From the point where the solution enters the filter, the rate at which the solution flows increases to 9 inches per second. There are 6 + 15 + 6 = 27 feet between the filter and the tank with the filtered solution. So, we can perform the following calculation and conversion: 27 ft × (1 sec/9 in) × (12 in/1 ft). Canceling the 9 and 27 leaves 3 × 12 = 36 seconds. Therefore, it takes 34 + 36 = **70 seconds** for the solution to travel from START to FINISH.

5.3 We can say that x milliliters of the 5% solution contain $0.05x$ milliliters of the active ingredient. In the 5-milliliter vials, this amount of active ingredient must account for 2% of the solution, or 0.02 × 5 = 0.1 milliliters. Therefore, $0.05x = 0.1$, so $x = 2$ milliliters. The amount of inactive ingredients added to each 5-milliliter vial, then, is 5 – 2 = **3 milliliters**.

5.4 If approximately 1 in 20 vials fails, this means 1/20 × 500 = 25 vials per day cannot be used. So, each manufacturing site will produce approximately 500 – 25 = 475 vials a day that can be used. To figure out the number of manufacturing sites needed to meet the current market demand of 1,500,000 vials a day, we calculate 1,500,000/475 ≈ 3157.89. Since 3157 manufacturing sites only produce 3157 × 475 = 1,499,575 vials, it follows that **3158 manufacturing sites** are needed to meet the market demand.

E-WEEK • MONDAY, FEB. 22 • ENVIRONMENTAL ENGINEERING

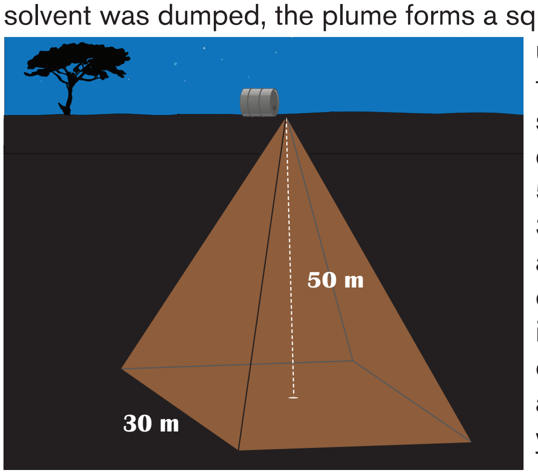
Environmental engineers work with other engineers, scientists and experts to control water and air pollution, as well as its impact on public health, waste disposal, recycling and more. *Environmental engineers at the Environmental Protection Agency (EPA) work to clean up some of the country's most contaminated toxic waste sites, called Superfund sites. At certain Superfund sites, where toxic waste has caused serious environmental issues,* ***groundwater****—water that is found underground in the gaps between layers of rock, soil or sand—is contaminated, which can lead to chemicals in recreational lakes, drinking water and wildlife habitats.*

1.1 The amount of contaminant in groundwater is reported using a measurement called parts per billion (ppb). This is calculated as:

$$\frac{\text{micrograms of contaminant}}{\text{micrograms of groundwater}} \times 10^9$$

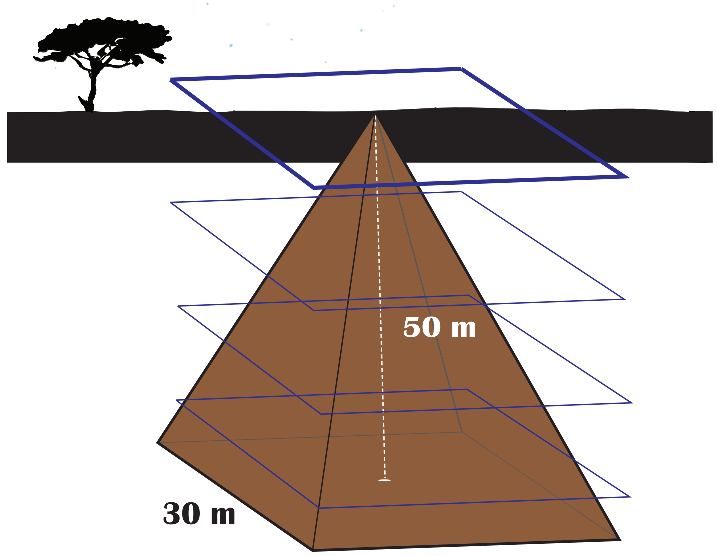
For groundwater to be considered safe, the amount of contaminant in the groundwater must be at or below 5 ppb. What is the maximum number of micrograms of contaminant that could be in 7×10^8 micrograms of safe groundwater?

1.2 A drum of dry-cleaning solvent, a toxic chemical, is dumped and seeps into the soil, contaminating the groundwater below. This space where contaminants have spread is called a **plume**. Exactly one year after the solvent was dumped, the plume forms a square pyramid underground that has spread to a depth of 50 m and 30 m across at its base edge. What is the volume of the plume after one year?



1.3 Each year after the first year, the depth of the plume in problem 1.2 triples and the base edge of the plume doubles. What is the volume of the plume exactly 4 years after the solvent was dumped?

1.4 When environmental engineers arrive to clean up the contaminated site referenced in problems 1.2 and 1.3, they must block off all the space above ground over any part of the plume to ensure the cleanup process is not disturbed. If the engineers begin cleanup exactly 4 years after the solvent was dumped, how many square meters must they block off *above* ground to cover the entire plume (in thick purple outlines below)?



FUTURE ENGINEERS

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E-WEEK • TUESDAY, FEB. 23 • SYSTEMS ENGINEERING

Systems engineers work in many different engineering fields. *They need expertise in both breadth and depth, often serving as both project manager and chief engineer. While more specialized engineers work on individual sub-systems and components of a large project, the systems engineer is needed to make sure they all work together to perform the overall desired function of the product.*

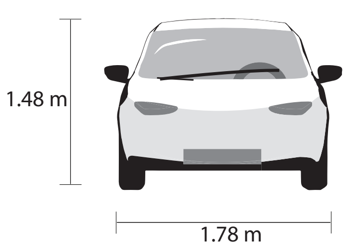
Kami is a systems engineer at a car manufacturer. Her team has been tasked with creating “a family sedan that rides like a sports car.” She has been working closely with the engineers designing the different components that all play a part in a car’s functioning, but she needs to test and improve the sedan’s aerodynamic efficiency to ensure their design will be aerodynamic enough to feel as fast as a sports car to consumers.

2.1 One measurement that contributes to a car’s aerodynamic efficiency is its **coefficient of drag** (C_d), which refers to the resistance of the car as it travels through the air. Typical sedans have a coefficient of drag between 0.34 and 0.50, but to meet the project goal of “a family sedan that rides like a sports car,” the C_d of the team’s sedan must be comparable to a typical sports car.

Based on the table at right showing the C_d of the 11 most popular sports car models, what is the median C_d of a typical sports car?

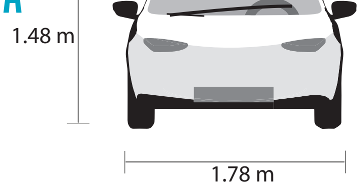
SPORTS CAR MODEL	COEFFICIENT OF DRAG
MC-1983	0.27
PI-227	0.314
THE PASCAL	0.377
MVC-II	0.31
THE PARALLEL	0.33
MC-2021X	0.363
THE OPEN ROAD	0.27
BREEZE 452 MLP	0.28
BB8-X	0.341
PI-R2	0.35
TRI-345	0.292

2.2 The other measurement that contributes to a car’s aerodynamic efficiency is its **frontal area** (A), or the total amount of space the car occupies when it is viewed from the front. This is calculated as 85% of height × width. The team’s first design, Sedan A, is shown at left. To the nearest thousandth, what is the frontal area of Sedan A?

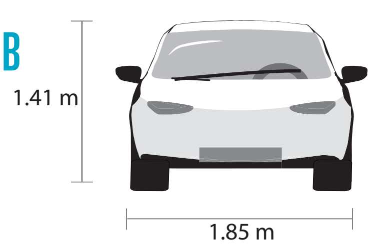


2.3 To compare the aerodynamics of two cars, engineers compare the value of the product of the coefficient of drag and the frontal area, or C_dA. The *lower* the value of C_dA, the *faster* the car. Kami asks her team for two more designs, Sedan B and Sedan C. All three designs have a C_d equal to what you calculated in 2.1. Which design should Kami choose if she wants the fastest sedan—Sedan A, Sedan B or Sedan C?

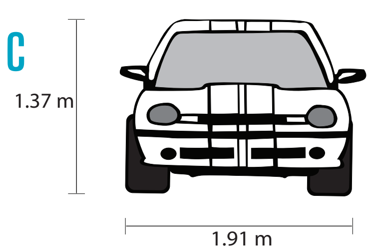
A



B



C

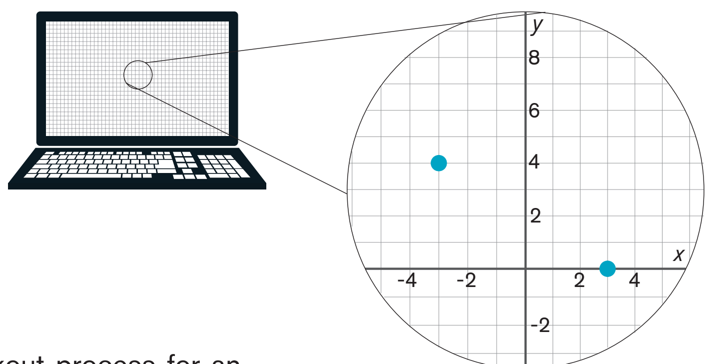


E-WEEK • WEDNESDAY, FEB. 24 • SOFTWARE ENGINEERING

***Software engineering is instrumental in all fields**, protecting and organizing user data, keeping business practices efficient and allowing for accessible communication and entertainment. The two main types of software engineers are applications software engineers—who design computer applications for users, such as games, word processors and internet browsers—and systems software engineers—who build full operating systems and networks for user-facing applications and integrate different software products onto single platforms.*

An **algorithm** is a set of rules to be followed during a problem-solving operation. The ability to understand, develop and apply a wide variety of algorithms is an important part of software engineering. The following problems look at some common algorithms found in the field.

3.1 In essence, a computer screen is a grid with various points filled in to create what you see on the screen...much like a coordinate plane. Software engineers use algorithms to "fill in" these points. Write an algorithm, in the form of an equation in slope-intercept form, that can be used to create a line that passes through the two points on the computer screen shown.



3.2 René is a software engineer working on the checkout process for an online store. When customers check out, the cost of shipping and tax must be added to the base price, *B*, of the items to be purchased to calculate the total cost, *T*. If tax is a percentage of the base price and shipping is a flat rate, use the table of values shown to write an equation that represents the algorithm René should use to calculate total cost on the store's website.

ITEM BASE PRICE (B)	TOTAL COST (T)
$25	$32.50
$50	$60
$75	$87.50
$100	$115

3.3 When someone sends a text message over a communications network, receivers detect if any errors occurred when the message was transmitted. A **checksum** is one common algorithm to do this: it identifies *if* an error occurred, but does not specify *which* error(s) and cannot catch every kind of error. For example, if numbers were assigned to every letter of the alphabet (A = 1, B = 2, ..., Z = 26), and someone sent a message that said OK, the values of these letters would be O = 15 and K = 11. The checksum algorithm would verify the message being received had a sum of 15 + 11 = 26 and, if the sum were *not* 26, would identify an error in the transmission. However, if the message above came through as KO, the sum would still equal 26, meaning the checksum would not catch this specific error.

Vanessa texts René the message BAD. Assigning the same numbers to the letters in the example above, and assuming all three-letter arrangements are equally likely, what is the probability that René receives a three-letter message that has an error the checksum does not catch? Express your answer as a common fraction.

E-WEEK • THURSDAY, FEB. 25 • AEROSPACE ENGINEERING

Aerospace engineering is all about designing and building machines that fly. *It is comprised of (1) aeronautical engineering, which focuses on aircraft, like jets and helicopters and (2) astronautical engineering, which is all about spacecraft, like satellites, spaceships and even spacesuits. Aerospace engineers must consider multiple factors to build aircraft and spacecraft, such as gravity, speed, fuel consumption, atmospheric pressure and conditions in space, so all types of math are commonly used in this field.*

To get a satellite into space, it must be attached to a rocket that can carry it there. Even if a satellite is small, the rocket will require a lot of fuel to lift it...and it will need *even more* fuel to lift that fuel. Engineers estimate how much fuel will be required with the **mass ratio**, or $m_{initial}/m_{final}$, where $m_{initial}$ is the initial mass of the rocket (including the rocket itself, the satellite and all its fuel) and m_{final} is the final mass of the rocket (after all its fuel has been burned).

A mass ratio of 15, for example, means the rocket will carry 15 times more fuel than the rest of the mass of the rocket. Typical mass ratios range from 8 to 20...in fact, for many launches, fuel makes up the majority of the initial mass!

4.1 To determine the mass ratio for a particular rocket launch, aerospace engineers use the **rocket equation**:

$$\text{mass ratio} = \frac{m_{initial}}{m_{final}} = 2.718^{\left(\frac{\text{change in velocity}}{\text{exhaust velocity}}\right)}$$

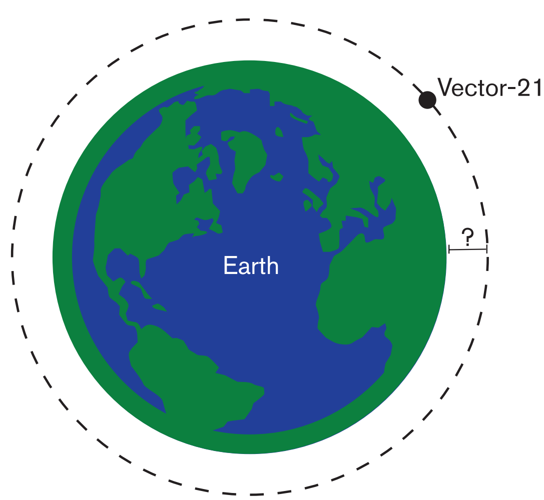
where *change in velocity*, the rocket's speed, equals the change in the rocket's position divided by the change in time and *exhaust velocity* is the rate at which exhaust gases leave the rocket's engine.

Coordinate Planes, Inc. launched a satellite, Vector-21, into orbit. The rocket carrying Vector-21 traveled 940,000 meters up in 158 seconds, before releasing the satellite. If the exhaust velocity was 2000 m/s, what is the mass ratio? Express your answer to the nearest tenth.

4.2 Fuel made up 91% of the initial mass ($m_{initial}$) in the launch of Vector-21. If the final mass (m_{final}) was 41,255 kg, how many kilograms of fuel were sent up? Express your answer to the nearest hundredth.

4.3 The Vector-21 satellite travels a total distance of 43,200 km to orbit Earth. Traveling at a rate of 8 km/s, how many minutes does it take the satellite to completely orbit Earth?

4.4 Given that Earth has a circumference of 40,075 km, how many kilometers above Earth's surface does the Vector-21 orbit? Express your answer to the nearest tenth.



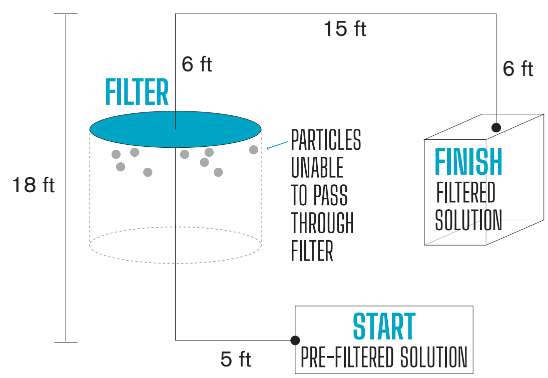
E-WEEK • FRIDAY, FEB. 26 • CHEMICAL ENGINEERING

Chemical engineers design and develop chemical manufacturing processes, and they do this in many different fields. *For example, they are vital in producing various types of fuel, pesticides, plastic and rubber products, foods...even some electronics! Right now, chemical engineers are instrumental in the production of COVID-19 vaccines around the world.*

Developing a vaccine requires many steps; some vary by specific vaccine, but certain steps must always take place. For example, towards the end of the vaccine manufacturing cycle, chemical engineers must use a filtration system to remove certain buffers from their solution. A **buffer** is a solution added to help maintain pH stability throughout the production of a medicine, but that is not intended to be in the final product.

5.1 Jorge, a chemical engineer, initially puts 700 liters of solution into the filtration system. During filtration, 12 liters do not get through the filter, 0.5 liter gets caught in the piping and 0.8 liter never makes it out of the initial container. By what percent does the volume of solution decrease during the filtration process? Express your answer to the nearest whole percent.

5.2 Until the solution leaves the filter, it flows through the system at a rate of 6 inches per second. After leaving the filter, the solution flows at a rate of 9 inches per second. How many seconds will it take the solution to travel from START to FINISH in the filtration system shown at right?



5.3 After filtration, a solution may need to be adjusted to ensure it has the right amount of **active ingredient**—the component of a medicine responsible for its effects. Jorge's team needs the filtered solution to be made up of 2% active ingredient. If, after filtration, the solution is made up of 5% active ingredient, how many milliliters of inactive ingredients must the engineers combine with the 5% solution to create a 5-milliliter vial of 2% solution?

5.4 A pharmaceutical company must produce 1,500,000 vials of vaccine per day, and a single manufacturing site produces 500 vials per day. If approximately 1 in 20 vials fails and cannot be used, what is the minimum number of manufacturing sites that can be expected to produce the required number of vials daily? Express your answer to the nearest whole number.