

2009-2010: How many scoring sequences can result in a total score of 4 points?

Start by determining how many combinations of four scores, each score being 2, 1 or 0, results in a sum of 4.

$$2, 2, 0, 0 \quad 2, 1, 1, 0 \quad 1, 1, 1, 1$$

Now let's determine how many sequences can be formed with each of the three combinations. When looking at the first sequence, any of the 4 scores (2, 2, 0, 0) could be first, any of the remaining 3 scores could be second, either of the remaining two scores could be third and the remaining score would be last. This might make you think that the total number of sequences for the first combination would be $4! = 24$; however, because the two 2s are indistinguishable from each other and the two 0s are indistinguishable from each other, we need to divide by $2! \times 2!$. Thus, the first combination (2, 2, 0, 0) has $4!/(2! \times 2!) = 6$ possible distinct sequences. Following similar logic to account for the duplicate 1s, the second combination (2, 1, 1, 0) has $4!/2! = 12$ possible sequences, and the third combination (1, 1, 1, 1) has $4!/4! = 1$ possible sequence. Thus, there are $6 + 12 + 1 = 19$ possible sequences that would result in Alli's final score being 4 points.