

MATHCOUNTS Fall 2016 Newsletter Poster Problem

What is the length of the **shortest** path **Pumpkin the Pig** can walk starting from **B** and walking along each segment **at least once** in equilateral triangle **ABC**?

Express your answer in simplest radical form.

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Since triangle **ABC** is equilateral, we know every side is equal, and the perimeter is $2\text{ ft} + 2\text{ ft} + 2\text{ ft} = 6\text{ ft}$. The segment **AD**, since it is perpendicular to side **BC**, is the height of the triangle and bisects side **BC** at **D**. Segments **BD** and **CD** are, therefore, each 1 ft.

We could use the Pythagorean Theorem to find the length of **AD**, but triangle **ADC** is a 30-60-90 right triangle. A property of 30-60-90 right triangles is that the longer leg has length $\sqrt{3}$ times that of the shorter leg. Therefore, **AD** is $\sqrt{3}$ ft. This means the distance **Pumpkin the Pig** must travel to eat all the strawberries is at least $6 + \sqrt{3}$.

However, there is no way to walk a path along every segment without retracing any segment if **Pumpkin** starts at point **B**. One segment must be retraced, so let's choose one of the shortest segments, **BD** or **DC**, to retrace. As shown on the next page, two such paths **Pumpkin** can walk are $B \rightarrow A \rightarrow C \rightarrow D \rightarrow B \rightarrow D \rightarrow A$ or $B \rightarrow D \rightarrow A \rightarrow C \rightarrow D \rightarrow B \rightarrow A$ (Note: there are more than the two ways to get the same path length).

Thus, the shortest length of a path **Pumpkin** can take walking along each segment at least once is $6 + \sqrt{3} + 1 = 7 + \sqrt{3}$ ft.

