MATHCOUNTS 2014–2015 HB Poster Problem

How many different 5-note sequences can the band FEEDBAG play using 5 of the 7 letters in their band name?

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We can think of a 5-note sequence as a 5-letter sequence. This problem is made more difficult because of the 2 Es in FEEDBAG. One way to come up with the number of sequences is to examine the various cases. Each possible 5-letter sequence will include 0, 1 or 2 Es.

Case 1: The only set of 5 letters that does not contain the letter E is FDBAG. But since these 5 letters can be arranged in 5! = 120 different orders, there are 120 different sequences consisting of these 5 letters.

Case 2: There are five combinations of 5 letters that contain 1 E. They are FEDBA, FEDBG, FEBAG, EDBAG and FEDAG. Again, each of these 5-letter combinations can be arranged in 5! = 120 different orders resulting in 5 × 120 = 600 different sequences.

Case 3: Finally, there are 10 combinations of 5 letters that contain 2 Es. They are FEEDB, FEEDA, FEEDG, FEEBA, FEEBG, FEEAG, EEDBA, EEDBG, EEDAG and EEBAG. Since 2 letters are the same in each of these 5-letter combinations, there are 5! ÷ 2 = 120 ÷ 2 = 60 different sequences of each 5-letter combination. That's 10 × 60 = 600 different sequences.

Thus, the total number of different 5-letter sequences is 120 + 600 + 600 = **1320** sequences.

Another way to think about this problem is to consider that the number of ways to arrange 5 of 7 objects is ${}_7P_5 = 7! \div 2! = 2520$ ways. But since the 2 Es in FEEDBAG are indistinguishable, this total figure counts the 600 sequences with 1 E twice. Similarly, this total figure counts the 600 sequences with 2 Es twice. Thus, the number of different 5-letter sequences is 2520 – 600 – 600 = **1320** sequences.