

MATHCOUNTS 2014–2015 Fall Newsletter Problem

At the exact moment Alejandro began to cross the 1432-ft Zakim Bridge from one end, cycling 28 ft/s, Bindi began to cross, running 12 ft/s from the opposite end. How much farther than Bindi had Alejandro traveled when they met?

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We will use the formula $d = r \times t$ (distance = rate \times time) and the relative speed of Alejandro and Bindi. Since they are moving from opposite ends of the bridge towards each other, at the time Alejandro and Bindi meet, they will have traveled a total of 1432 feet. The relative speed for Alejandro and Bindi to travel this distance is their combined speeds $28 + 12 = 40$ ft/s. In other words, for every second Alejandro and Bindi spend crossing the bridge, they are traveling 40 feet out of the total 1432 feet. By substituting these two values into the equation, we can determine how many seconds Alejandro and Bindi each traveled before meeting. Doing so yields $1432 = 40t$, thus $t = 35.8$ seconds. At 28 ft/s, Alejandro will have traveled $d = 28 \times 35.8 = 1002.4$ feet. At 12 ft/s, Bindi will have traveled $d = 12 \times 35.8 = 429.6$ feet. The difference in these two distances is $1002.4 - 429.6 = \mathbf{572.8}$ feet.

Another approach is to use ratios and proportional reasoning. The ratio of the rates of Alejandro and Bindi is $28/12 = 7/3$. That means Alejandro will have covered $7/10$ of the total distance and Bindi will have covered the other $3/10$. That's a difference of $7/10 - 3/10 = 4/10 = 2/5$. And $2/5$ the total distance across the bridge is $(2/5) \times 1432 = \mathbf{572.8}$ feet.