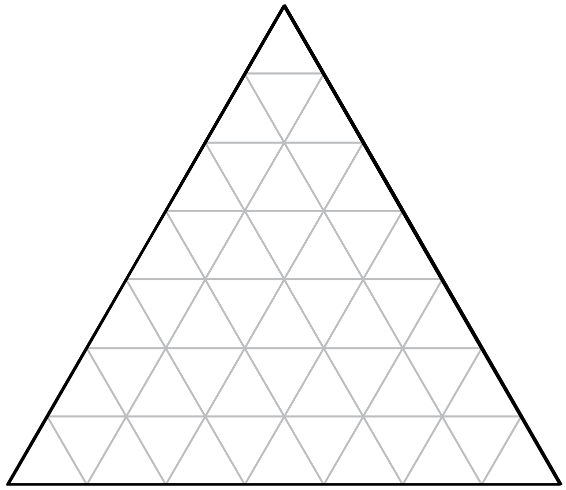
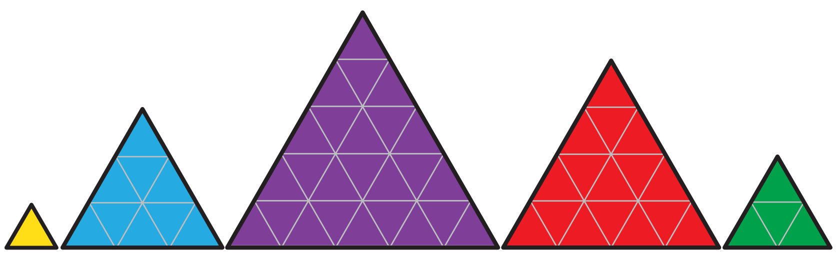
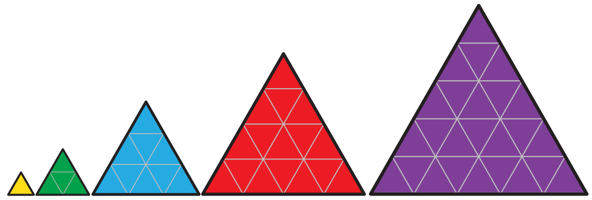
MATHCOUNTS 2013-2014 HB Poster Problem



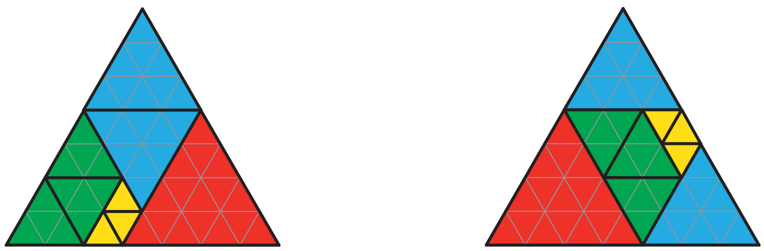
What is the fewest number of non-overlapping triangles, in the five sizes below, needed to completely cover this equilateral triangle?



MATHCOUNTS 2013-2014 HB Poster Problem SOLUTION

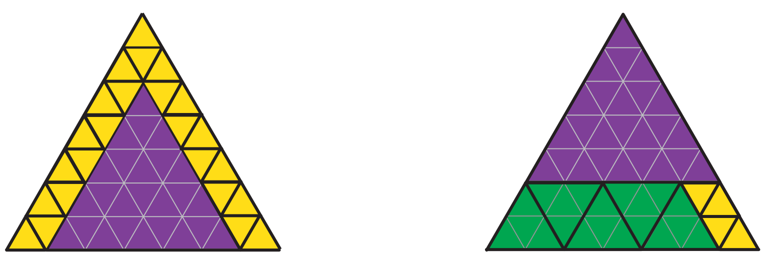


For simplicity, let's think of the five smaller triangles as yellow, green, blue, red and purple triangular tiles, as shown. The equilateral triangle can be completely covered using no fewer than **9** tiles. Below are two possible solutions, each using three yellow tiles, two blue tiles, one red tile and three green tiles.

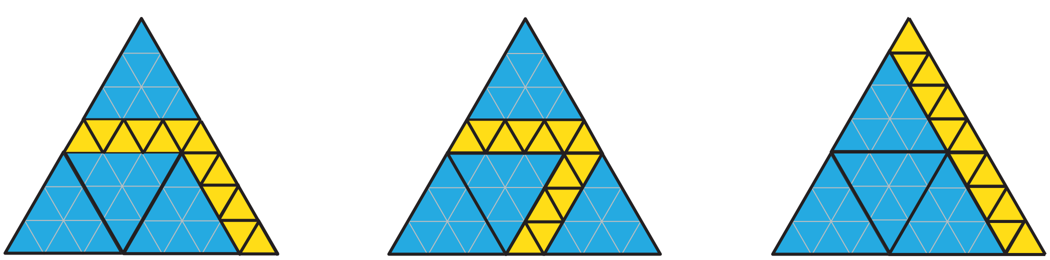


Now we'll show why it's not possible to completely cover the given equilateral triangle with fewer than 9 tiles. Let's assume that it can be covered in 8 or fewer tiles. Consider the following cases:

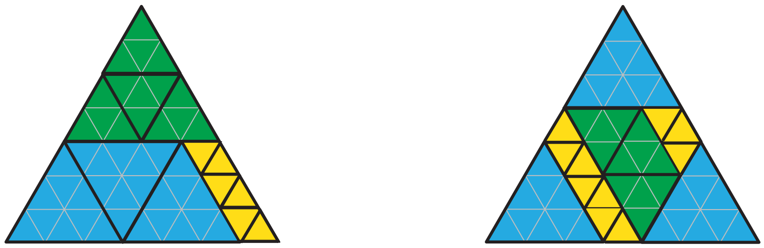
Case 1: Using a purple tile, the remainder of the equilateral triangle can be covered by 24 yellow tiles, as shown in the first figure below, or a combination of yellow tiles and green tiles. As the second figure below shows, at most five green tiles can be used, leaving the remainder of the equilateral triangle to be covered by four yellow tiles. That's a total of 10 tiles. Since any other arrangement that includes a green tile requires a total of more than 10 tiles to cover the equilateral triangle, we can conclude that there is no arrangement using a green tile that works and has 8 or fewer tiles in total.



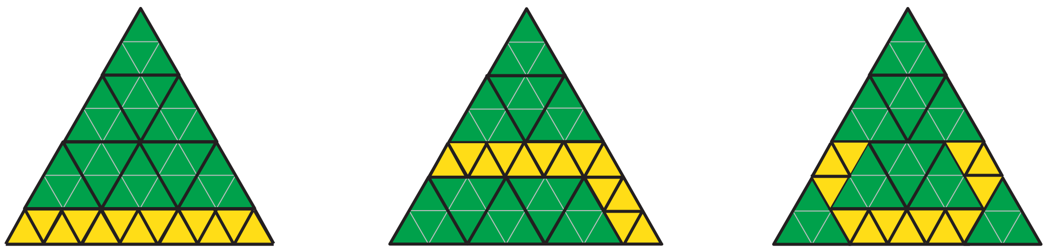
Case 2: Since the solutions above show that 9 is the minimum number of tiles that will completely cover the equilateral triangle when using a red tile, the next case we will consider uses blue tiles or smaller. As shown below, when four blue tiles are used, the only way to completely cover the remainder of the equilateral triangle is by using thirteen yellow tiles. That's a total of 17 tiles.



Case 3: As the first figure below shows, when three blue tiles are used with four green tiles, the remainder of the equilateral triangle can be completely covered with six yellow tiles. That's a total of 13 triangles. The second figure shows that when three blue tiles are used with three green tiles, the remainder of the equilateral triangle can be completely covered using ten yellow tiles. That's a total of 16 tiles.



Case 4: As shown below, when nine green tiles are used, the remainder of the equilateral triangle can be completely covered using thirteen yellow tiles.



Clearly, the equilateral triangle cannot be completely covered in fewer than 9 tiles when using only yellow tiles.

We can conclude that completely covering the equilateral triangle in 8 or fewer tiles is not possible. Therefore, the equilateral triangle can be completely covered using no fewer than **9** non-overlapping triangles in the five sizes shown.