

MATHCOUNTS 2011-2012 HB Poster Problem

On planet Forux there are Forisies and Forlegies. Every Forisie has exactly 4 eyes and exactly 2 legs. Every Forlegie has exactly 4 legs and exactly 1 eye. A gathering of Forisies and Forlegies has an equal number of eyes and legs. What is the minimum number of aliens present?

Solution

Consider a gathering of one Forisie with 4 eyes and 2 legs and one Forlegie with 1 eye and 4 legs. The total number of eyes is $4 + 1 = 5$, and the total number of legs is $2 + 4 = 6$. Right away we notice that a gathering of any size of Forisies and Forlegies will have an even number of legs because the total number of Forisie legs will always be a multiple of 2, and the total number of Forlegie legs will always be a multiple of 4. Therefore, to ensure we also have an even number of eyes, we'll need to have an even number of Forlegies. So the gathering must have at least two Forlegies. Let's consider a gathering of one Forisie and two Forlegies. In this case, the total number of eyes is $1(4) + 2(1) = 4 + 2 = 6$, and the total number of legs is $1(2) + 2(4) = 2 + 8 = 10$. We need more eyes at our gathering. If we add one more Forisie we have a total of $2(4) + 2(1) = 8 + 2 = 10$ eyes and a total of $2(2) + 2(4) = 4 + 8 = 12$ legs. We still need more eyes, so let's try adding another Forisie to the gathering. We now have three Forisies and two Forlegies with a total of $3(4) + 2(1) = 12 + 2 = 14$ eyes and a total of $3(2) + 2(4) = 6 + 8 = 14$ legs. A gathering of three Forisies and two Forlegies has an equal number of eyes and legs. That means the minimum number of aliens present is $3 + 2 = 5$ aliens.

You also can think of this problem algebraically. Let x represent the number of Forisies and y be the number of Forlegies. Since each Forisie has exactly four eyes and each Forlegie has exactly one eye, the total number of eyes in a gathering of these aliens is equal to $4x + y$. Similarly, since each Forisie has exactly two legs and each Forlegie has exactly four legs, the total number of legs in a gathering of these aliens equals $2x + 4y$. We want to know the minimum number of aliens required for the number of eyes and the number of legs to be equal. In other words, what is the ratio of x to y when $4x + y = 2x + 4y$? Combining like terms we get $4x - 2x = 4y - y \rightarrow 2x = 3y$. Then dividing each side by y and 2 yields $x/y = 3/2$. So, if the number of eyes and legs is equal in a gathering of aliens, there must be three Forisies for every two Forlegies. It follows that the minimum number of aliens needed to accomplish this equality is three Forisies and two Forlegies. That means the minimum combined total of these alien species needed is $3 + 2 = 5$ aliens.