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**MATHCOUNTS Newsletter Poster Problem**

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Abe has a bag containing 340 coins and 160 bills totaling \$1,809. There are pennies, nickels, fives, tens and twenties. The total value of the tens is equal to the total value of the twenties. There are the same number of fives and twenties. What is the total value of the coins and bills bearing Lincoln's portrait?

**Solution**

Let's assign the quantity each monetary denomination in the bag a variable as follows:

number of pennies =  $P$

number of nickels =  $N$

number of fives =  $F$

number of tens =  $T$

number of twenties =  $W$

Next, let's write some equations to illustrate the relationships provided.

- Since the bag contains a total of 500 pieces of money (bills and coins) we have  $P + N + F + T + W = 500$  (1).
- The total value of all the coins in the bag is \$1,809 so  $(0.01)P + (0.05)N + (5)F + (10)T + (20)W = 1,809$  (2).
- We are also told that the value of the tens and the value of the twenties in the bag are equivalent. Thus,  $10T = 20W \rightarrow T = 2W$  (3).
- Also,  $F = W$  (4) since there are an equal number of fives and twenties in the bag.
- Lastly, we know that there are 340 coins in the bags, which means  $P + N = 340$  (5).

Using (3) and (4) to substitute for  $W$  in (1) and simplifying yields  $P + N + W + 2W + W = 500 \rightarrow P + N + 4W = 500$ . Then using (5) to substitute for  $P + N$  and simplifying results in the equation  $340 + 4W = 500 \rightarrow 4W = 160 \rightarrow W = 40$ . That means that  $T = 2(40) \rightarrow T = 80$  and  $F = 40$ . Next we substitute the values of  $W$ ,  $T$  and  $F$  in (2) and simplify to get  $(0.01)P + (0.05)N + (5)(40) + (10)(80) + (20)(40) = 1,809 \rightarrow (0.01)P + (0.05)N + 1,800 = 1,809 \rightarrow (0.01)P + (0.05)N = 9$  (6). Now we have a system of equations (5) and (6) that we can solve for  $P$  and  $N$ . Multiplying (6) by  $-100$  yields  $-P - 5N = -900$ . Adding this new equation to (5) and simplifying the result is  $-4N = -560 \rightarrow N = 140$ . Substituting for  $N$  in (5) and simplifying we that  $P + 140 = 340 \rightarrow P = 200$ . Since the pennies and fives bear Lincoln's portrait the total value of those denominations is  $200(0.01) + 40(5) = \mathbf{\$202.00}$ .