MATHCOUNTS®

2021 State Competition
Sprint Round Problems 1–30

Name _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

1. _____ candy bars

Holly is selling candy bars to raise money for her softball team. She starts with 80 candy bars and sells 32 to her neighbors and 15 to her grandparents. How many candy bars does Holly have left to sell?

2. _____ times

A recipe for chocolate chip cookies calls for $2\frac{1}{4}$ cups of chocolate chips. Jaime only has a $\frac{1}{4}$-cup measuring cup. To measure the exact amount of chocolate chips needed for the recipe, how many times does Jaime need to fill up the measuring cup?

3. _____

If 0.0036 divided by n is equal to 0.000012, what is the value of n?

4. _____

Alonzo draws a diagonal of a convex polygon with 8 sides. The diagonal divides the polygon into two smaller polygons, one with 6 sides and one with n sides. What is the value of n?

5. _____

When $x = 3$, what is the value of the expression 4^{x-5}? Express your answer as a common fraction.

6. _____ feet

Sylvia the snail crawls 23 inches in the first hour, then 59 inches in the second hour, then 49 inches in the third hour. How many feet does Sylvia crawl in these three hours? Express your answer to the nearest whole number.

7. _____ seashells

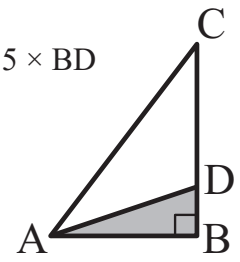
Ben, Rachel and Teri collected seashells on a beach. Ben collected five more than twice as many seashells as Teri collected. Rachel collected seven less than four times as many seashells as Teri collected. If Ben and Rachel collected the same number of seashells, how many seashells did Teri collect?

8. $ _____

At Kickin' Chicken, a chicken sandwich costs $4, plus 7% sales tax. If Quincy orders a chicken sandwich and pays with a $20 bill, how much change will he receive?

9. _____ in^2

In right triangle ABC, point D is on side BC, as shown. If $BC = 5 \times BD$ and the area of ΔABD is 8 in^2, what is the area of ΔABC?



10. _____ dolls

By selling handmade dolls, Hannah hopes to earn at least $1500 to donate to the local hospital. If she makes between $40 and $90 for each doll she sells, what is the absolute difference between the minimum and maximum number of dolls she must sell to meet her minimum fundraising goal?

11. $\$$ _____ A fence is to be placed along the perimeter of a rectangular field with area 400 ft^2 and minimum possible perimeter. If one foot of fencing costs \$2.50, what is the total cost of the fencing needed to completely enclose the field?

12. _____ cm If each side of a square is decreased in length by 2 cm, its area is decreased by 160 cm^2. What is the original side length of the square?

13. _____ In Mr. Patterson's class, the average score among students who studied for an exam was 78. The average among students who did not study was 54. The overall class average was 70. What portion of the class did not study? Express your answer as a common fraction.

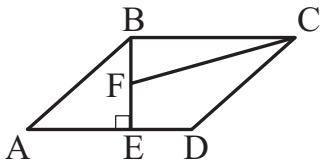
14. _____ Forty balls, numbered 1 to 40, are placed in a bag. What is the probability that the number on a randomly drawn ball is a multiple of 4 or 5? Express your answer as a common fraction.

15. _____ If $3^{(3^n)} = 27^{(27^{27})}$, what is the value of n?

16. _____ Paloma has a bag containing red, blue and white marbles. The ratio of red to blue marbles is 4:3, and the ratio of blue to white marbles is 7:2. What is the probability of Paloma randomly drawing a blue marble from this bag? Express your answer as a common fraction.

17. _____ people Riley's entire extended family goes out to eat. Each person orders either a salad for $6.50 or a cheeseburger for $7.50. They spend a total of $138.00 and buy four more salads than cheeseburgers. How many people, including Riley, are in the family?

18. _____ ABCD is a rhombus with side length 6. The measure of angle ABC is 150 degrees. Segment BE is perpendicular to base AD, and F is the midpoint of segment BE. The length of segment CF, expressed as a common fraction in simplest radical form, is $\frac{a\sqrt{b}}{c}$. What is the value of $a + b + c$?



19. _____ values When 257 is divided by m, the remainder is 5. How many possible positive integer values are there for m?

20. _____ games In Pierre's sports league, each team plays at most one game per day. Furthermore, no team is allowed to play games on three consecutive days, nor may any team play four or more games in any five consecutive days. Under these constraints, what is the maximum number of games Pierre's team could play in a 108-day interval?

21. _____ What is the value of the expression shown? Express your answer as a common fraction.

$$3+\cfrac{3}{3+\cfrac{3}{3+\cfrac{3}{3+\cfrac{3}{3+\frac{3}{3}}}}}$$

22. _____ ordered pairs The equation $x^2 + y^2 - 2x + 4y = -4$ is true for how many ordered pairs of integers (x, y)?

23. _____ If n is an integer such that the value of the expression $4n + 5n + 6n + 7n$ is a four-digit positive integer containing each of the digits 4, 5, 6 and 7, what is the greatest possible value of n?

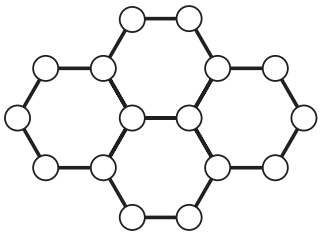
24. _____ If x and y are positive numbers with $x - y = 12$ and $\sqrt[3]{x} - \sqrt[3]{y} = 2$, what is the value of xy? Express your answer as a common fraction.

25. _____ In a certain game, Players 1 through 5 take turns rolling a standard six-sided die, beginning with Player 1. If a six is rolled, that player wins. If any other number is rolled, the die is given to the player whose number corresponds to the number rolled, even if that is the player whose turn it currently is. What is the probability that Player 1 wins? Express your answer as a common fraction.

26. _____ If t is a real number such that $t + \frac{1}{t} = 3$, what is the value of $t^5 + \frac{1}{t^5}$?

27. _____ In base b, we have $r = 0.\overline{57}_b$ and $3r = 1.\overline{06}_b$. What is the value of r in base ten? Express your answer as a common fraction.

28. _____ The integers from 1 to 16 are placed in the 16 circles in the figure shown, with each number occurring exactly once and so that the sum of the six numbers around each hexagon is S. What is the greatest possible value of S?



29. _____ Marian throws a dart that lands randomly on a dartboard shaped like an isosceles trapezoid with side lengths 12 inches, 12 inches, 12 inches and 24 inches. What is the probability that the dart is closer to the 24-inch side than it is to any of the other three sides of the dartboard? Express your answer as a common fraction.

30. _____ For $0 \le x \le 1$, the function $f(x)$ satisfies the relations $f\left(\frac{x}{x+1}\right) = \frac{f(x)}{2}$ and $f(1 - x) = 1 - f(x)$. What is the value of the expression $f\left(\frac{2}{3}\right) + f\left(\frac{2}{5}\right) + f\left(\frac{2}{7}\right) + \cdots + f\left(\frac{2}{2n+1}\right) + \cdots$? Express your answer as a common fraction.