

MATHCOUNTS®

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2021 STATE COMPETITION Target Round Problems 1–8

Name _____

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DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.



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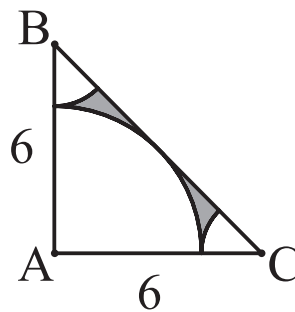
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1. _____ If a , b and c represent the distinct numbers 1, 2 and 3, in some order, what is the greatest possible value of the expression $817 \times a + 512 \times b + 210 \times c$?

2. _____ A year ago, Karen grew some popcorn. She removed kernels from the cob and put them in containers. She had 14 containers each holding 6 pounds $9\frac{5}{8}$ ounces and one container holding 4 pounds $7\frac{3}{4}$ ounces. In all, she had x pounds y ounces of popcorn kernels. If $y < 16$, what is the value of $x + y$? Express your answer as a decimal to the nearest tenth.

3. apples Mira's parents planted an apple tree when she was born. In the year Mira turned 8 years old, the tree started bearing fruit. That year, there were 48 apples. The number of apples increased by 50% every year after that. How many apples grew on the tree in the year Mira turned 12 years old?

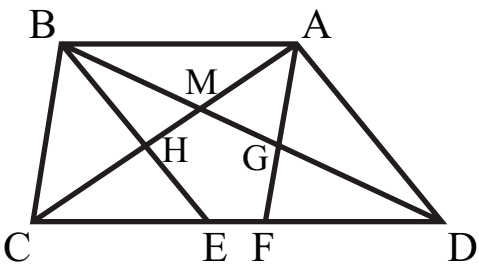
4. cm^2 Right triangle ABC has $AB = AC = 6$ cm. Circular arcs are drawn with centers at A, B and C, so that the arc centered at A is tangent to side BC and so that the arcs centered at B and C are tangent to the arc centered at A, as shown. What is the area of the shaded region inside $\triangle ABC$ but outside the three arcs? Express your answer as a decimal to the nearest hundredth.



5. _____ Some numbers are hidden in this sequence of positive integers, which contains each integer from 1 to 13 exactly once. Every term after the first term is 1 more than the previous term, 2 more than the previous term, or 3 less than the previous term. What is the product of the two numbers represented by A and B?

1, ■, A, 6, ■, ■, 7, B, ■, 12, ■, ■, 13

6. _____ ft² ABCD is a trapezoid with parallel bases of AB = 8 feet and CD = 14 feet. Point M is the intersection of the diagonals of trapezoid ABCD. Points E and F lie on base CD to form parallelograms ABCF and ABED. If the area of trapezoid ABEF is 60 ft², what is the area of triangle ADM? Express your answer as a common fraction.



7. _____ The parabola $y = x^2 - x - 2$ and the line $y = x + 1$ intersect in two points. What is the distance between these two points? Express your answer in simplest radical form.

8. _____ Not including the identity transformation, the eleven transformations that preserve the regular hexagon shown are counterclockwise rotations by 60° , 120° , 180° , 240° and 300° and reflections across the six dashed lines shown. Kristina randomly picks six transformations T_1, T_2, T_3, T_4, T_5 and T_6 , with replacement, from this set of eleven. She performs these six transformations on the hexagon, in succession. The probability that the point P is transformed to each of the hexagon's six vertices exactly once during this process is $\frac{k}{11^6}$. What is the value of k ?

