Sprint 1

The number of candy bars that Holly has left is $80 - (32 + 15) = 80 - 47 = \mathbf{33}$ candy bars.

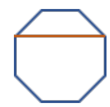
Sprint 2

To measure out exactly 2¼ cups of chocolate chips, Jaime needs to fill the ¼-cup measuring cup $\frac{2\frac{1}{4}}{1/4} = \frac{9\not{/4}}{1\not{/4}} = \mathbf{9}$ times.

Sprint 3

The problem text translates to the equation $\frac{0.0036}{n} = 0.000012$, which implies that $n = \frac{0.0036}{0.000012}$. Multiplying the numerator and denominator by 10^6 moves the decimal point of each number six digits to the right, and we get $\frac{3600.}{12.} = \mathbf{300}$.

Sprint 4



The added diagonal is to be 1 side out of a new 6-sided polygon, so the other 5 sides must come from the original 8-sided polygon; the remaining 3 sides of the original 8-sided polygon combine with the diagonal to form a **4**-sided polygon.

Sprint 5

Substituting 3 for x in the expression 4^{x-5} gives us $4^{3-5} = 4^{-2} = \frac{1}{4^2} = \mathbf{\frac{1}{16}}$.

Sprint 6

Sylvia crawls a total of $23 + 59 + 49 = 131$ inches, which is equivalent to $131 \not{\text{in}} \times \frac{1 \text{ ft}}{12 \not{\text{in}}} =$ $10\ \frac{11}{12}$ feet. That rounds to **11** feet.

Sprint 7

From the problem statement, we can write the equations $B = 2T + 5$, $R = 4T - 7$ and $B = R$. Since $B = R$, we can set the expressions for B and R equal to each other to get $2T + 5 = 4T - 7$. Simplifying and solving for T, we get $12 = 2T$, so $T = 6$. So, Teri collected **6** seashells.

Sprint 8

The amount of change Quincy will receive is $\$20 - \$4 \times (1 + 0.07) = \$20 - \$4 \times 1.07 = \$20.00 - \$4.28 = \$\mathbf{15.72}$.

Sprint 9

The area enclosed by △ABC is $\frac{1}{2}\ \text{AB} \cdot \text{BC} = \frac{1}{2}\ \text{AB} \cdot (5 \times \text{BD}) = 5 \times \left(\frac{1}{2}\ \text{AB} \cdot \text{BD}\right)$, which is 5 times the area enclosed by △ABD. Thus, the area of △ABC is $5 \times 8\ \text{in}^2 = \mathbf{40}\ \text{in}^2$.

Sprint 10

Let d represent the number of dolls sold. To make at least \$1500 at \$40 per doll requires that $d \geq \frac{\$1500}{\$40} = 37.5$. So, Hannah must sell at least 38 dolls at this price. To make at least \$1500 at \$90 per doll requires that $d \geq \frac{\$1500}{\$90} = 16.\overline{6}$. So, Hannah must sell at least 17 dolls at this price. The absolute difference in the minimum and maximum number of dolls she must sell is $38 - 17 = \mathbf{21}$ dolls.

Sprint 11

The minimum possible perimeter of a rectangle enclosing a specific area occurs when the rectangle is a square of side length s. The perimeter is $4s$ and the area is s^2, so the minimum perimeter is 4 times the square root of the area. For an area of 400 ft^2 the minimum perimeter is $4\sqrt{400\ \text{ft}^2} = 4 \times 20\ \text{ft} = 80\ \text{ft}$. At \$2.50 for each foot, that is a total cost of $80 \times \$2.50 = \$\mathbf{200}$ or $\$\mathbf{200.00}$.

Sprint 12

Let x represent the original side length of the square. Based on the information given, we know that $160 = x^2 - (x-2)^2 = x^2 - (x^2 - 4x + 4) = 4x - 4$. Thus, $164 = 4x$, and $x = 41$. So, the original side length of the square is $\mathbf{41}$ cm.

Sprint 13

Let c represent the total number of students in the class, and let n represent the number of students who did not study for the exam. Based on the average scores given, we know that $54n + 78(c - n) = 70c$. Simplifying, we get $54n + 78c - 78n = 70c \rightarrow 78c - 24n = 70c \rightarrow 8c = 24n$. So, the ratio of students who did not study to the total number of students in the class is $n/c = 8/24 = \mathbf{1/3}$. Alternatively, the value 70 is $\frac{70-54}{78-54} = \frac{16}{24} = \frac{2}{3}$ of the way from 54 to 78. Thus, $\frac{2}{3}$ of the students are in the group whose average score was 78, and the group whose average score was 54 has the remaining $\mathbf{1/3}$ of the students who did not study.

Sprint 14

From 1 to 40, inclusive, there are 10 multiples of 4 (4, 8, 12, 16, 20, 24, 28, 32, 36, 40); there are 8 multiples of 5 (5, 10, 15, 20, 25, 30, 35, 40). But 2 of those 10 + 8 = 18 numbers are multiples of both 4 and 5 (20 and 40). Therefore, 18 − 2 = 16 of the 40 balls have a number that is a multiple of 4 or 5, resulting in a probability of $16/40 = \mathbf{2/5}$. Alternatively, let $S = \{n\colon 1 \leq n \leq 40 \text{ and } 4|n\}$ and $T = \{n\colon 1 \leq n \leq 40 \text{ and } 5|n\}$, where n is assumed to be an integer. [Note the notation $m|n$ means that m divides n, or, equivalently, n is a multiple of m.] $|S \cup T| = |S| + |T| - |S \cap T|$. There are 10 multiples of 4 in 1 through 40 and 8 multiples of 5 in 1 through 40. The elements of $S \cap T$ are those integers from 1 through 40 that are divisible by both 4 and 5, that is, divisible by $\text{LCM}(4, 5) = 20$, of which there are 2 values. Thus, the desired count is $10 + 8 - 2 = 16$ out of a total count of 40, for a probability of $16/40 = \mathbf{2/5}$.

Sprint 15

Because $27 = 3^3$, we have $27^{27^{27}} = (3^3)^{(3^3)^{27}} = 3^{3(3^{3\times 27})} = 3^{3(3^{81})} = 3^{3^{82}}$, so $n = \mathbf{82}$.

Sprint 16

Let R, B and W represent the numbers of red, blue, and white marbles, respectively. We are told that $\frac{R}{B}=\frac{4}{3}$ and $\frac{B}{W}=\frac{7}{2}$. To have blue appearing as 3 and as 7 in two integer ratios, B must be a multiple of the least common multiple of 3 and 7, thus a multiple of 21. Use $B=21$. Then, to achieve the desired ratios, $R=28$ and $W=6$. Thus, we have 21 blue marbles out of $21+28+6=55$ marbles in total, for a probability of $\mathbf{\frac{21}{55}}$.

Sprint 17

Let S represent the number of salads ordered, and let C represent the number of cheeseburgers ordered. We can write the following two equations: $138.00=6.50S+7.50C$ and $S=C+4$. Substituting $C+4$ for S in the first equation gives us $138=6.5(C+4)+7.5C$. Simplifying and solving for C, we get $138=14C+26$, so $112=14C$, and $C=8$. Therefore, $S=8+4=12$ and $C+S=8+12=20$. That means there are **20** people in the family.

Sprint 18

Since ∠A and ∠ABC are supplementary, it follows that $m\angle A=180°-m\angle \text{ABC}=180°-150°=30°$. Now, △ABE is a right triangle with hypotenuse AB = 6, and the length of the leg opposite ∠A of measure 30° is 1/2 the length of the hypotenuse, so BE = 3. We are told that F is the midpoint of $\overline{\text{BE}}$, so $\text{BF}=\frac{1}{2}\text{BE}=\frac{3}{2}$. As a side of the rhombus, BC = 6 and is perpendicular to BF. Applying Pythagorean theorem for right triangle BCF, we have $\text{CF}=\sqrt{(\text{BC})^2+(\text{BF})^2}=\sqrt{6^2+\left(\frac{3}{2}\right)^2}=\sqrt{36+\frac{9}{4}}=\sqrt{\frac{144+9}{4}}=\sqrt{\frac{153}{4}}=\sqrt{\frac{9\times 17}{4}}=\frac{3\sqrt{17}}{2}$. This is of the form $\frac{a\sqrt{b}}{c}$, with $a=3$, $b=17$ and $c=2$. So, $a+b+c=3+17+2=\mathbf{22}$.

Sprint 19

We know that 257 leaves a remainder of 5 when divided by m, and 5 less than 257 is 252. So, 252 divided by m leaves a remainder of 0, meaning m is a divisor of $252=2^2 3^2 7^1$, and 252 has $(2+1)(2+1)(1+1)=18$ divisors. The divisors 1, 2, 3, and 4 do not work because a division by them cannot leave a remainder of 5, so there remain **14** satisfactory values for m.

Sprint 20

Let's not waste game days, so let's use the first 2, and we must skip day 3 to avoid 3 in a row. Then, we'll have a game on day 4, and we must skip day 5 to avoid more than 3 games in 5 days. We can repeat this 5-day pattern GG_G_ over and over without violating any restrictions. In 108 days, the full pattern can occur 21 times with 3 days leftover. At most, 2 games can be played in those 3 days. That yields a maximum total of $21\times 3+2=\mathbf{65}$ games.

Sprint 21

Applying the order of operations, we have $3+\frac{3}{3+\frac{3}{3+\frac{3}{3+\frac{3}{3+\frac{3}{3}}}}}=3+\frac{3}{3+\frac{3}{3+\frac{3}{3+\frac{3}{4}}}}=3+\frac{3}{3+\frac{3}{3+\frac{3}{15/4}}}=3+\frac{3}{3+\frac{3}{3+\frac{4}{5}}}=$

$3+\frac{3}{3+\frac{3}{19/5}}=3+\frac{3}{3+\frac{15}{19}}=3+\frac{3}{72/19}=3+\frac{19}{24}=\mathbf{\frac{91}{24}}$.

Sprint 22

Rearranging $x^2 + y^2 - 2x + 4y = -4$ to put x terms together and y terms together yields $x^2 - 2x + y^2 + 4y = -4$. Next, we can complete squares by adding $(-2/2)^2 = 1$, since $x^2 - 2x + 1 = (x-1)^2$ and by adding $(4/2)^2 = 4$, since $y^2 + 4y + 4 = (y+2)^2$. Because we've added 1 + 4 = 5 to the left side of the equation, we need to add 5 to the right side as well, to get $(x-1)^2 + (y+2)^2 = 1$, which is the equation of a circle of radius 1, centered at $(1, -2)$. The center is at an ordered pair of integers, so the only points that are ordered pairs of integers lying on the circle are 1 left from center $(0, -2)$, 1 right from center $(2, -2)$, 1 up from center $(1, -1)$, and 1 down from center $(1, -3)$, for a total of **4** ordered pairs.

Sprint 23

Let $4n + 5n + 6n + 7n = 22n = 1000\text{A} + 100\text{B} + 10\text{C} + \text{D}$, the desired number, which will be abbreviated as ABCD, each letter representing one digit that is 4, 5, 6, or 7. Divisibility by 22 requires divisibility by 2 and 11: by 2 requires D to be 4 or 6; for the specified mix of digits, by 11 requires D – C + B – A = 0. This necessitates D + B = 11 and C + A = 11. Because D must be 4 or 6, that requires B to be 7 or 5, respectively. For the value to be as large as possible, A needs to be the greatest digit, 7, requiring C = 4, so D = 6 and B = 5. Thus, $22n = 7546$ and $n = \mathbf{343}$.

Sprint 24

Let $\sqrt[3]{x} = a$ and $\sqrt[3]{y} = b$. So, $x = a^3$ and $y = b^3$. From the given information, we know that $a^3 - b^3 = 12$ and $a - b = 2$. We also know that $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. That means $(a - b)(a^2 + ab + b^2) = 12$. Substituting 2 for $a - b$, we have $2(a^2 + ab + b^2) = 12$, so $a^2 + ab + b^2 = 6$. Now, since $a - b = 2$, it follows that $(a - b)^2 = 2^2$, and $a^2 - 2ab + b^2 = 4$. Subtracting the equations $a^2 + ab + b^2 = 6$ and $a^2 - 2ab + b^2 = 4$ yields $3ab = 2$, so $ab = 2/3$. Therefore, $xy = a^3b^3 = (ab)^3 = (2/3)^3 = \mathbf{8/27}$.

Alternatively, we have $8 = 2^3 = \left(\sqrt[3]{x} - \sqrt[3]{y}\right)^3 = x - 3\sqrt[3]{x^2y} + 3\sqrt[3]{xy^2} - y = (x - y) - 3\left(\sqrt[3]{x} - \sqrt[3]{y}\right)\sqrt[3]{xy} = 12 - 3(2)\sqrt[3]{xy}$. Thus, $4 = 6\sqrt[3]{xy}$, and $\sqrt[3]{xy} = \frac{2}{3}$, so $xy = \frac{2^3}{3^3} = \mathbf{\frac{8}{27}}$.

Sprint 25

Player 1 wins outright, with no opportunity for any other player to play, by rolling a 6 on the first roll, with probability $\frac{1}{6}$. For any other outcome of the first roll, with a total probability of $\frac{5}{6}$, each player has equal probability to play and equal probability to win. Thus, each player, including Player 1, can get to a second roll in the game and ultimately win with probability $\frac{5}{6} \times \frac{1}{5} = \frac{1}{6}$. Winning on the first roll and winning on a later roll are mutually exclusive events, so the probability of one or the other occurring for Player 1 is the sum of the two probabilities: $\frac{1}{6} + \frac{1}{6} = \mathbf{\frac{1}{3}}$.

Sprint 26

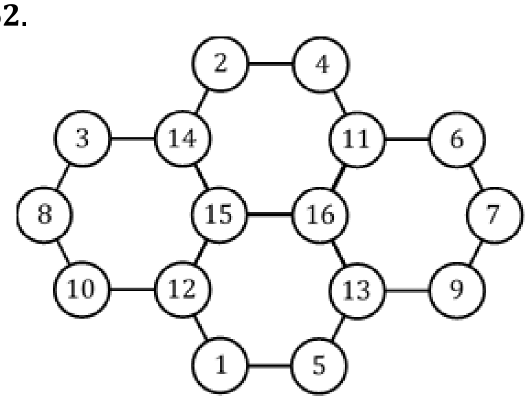
We are told that $t + \frac{1}{t} = 3$. So, it follows that $\left(t + \frac{1}{t}\right)^2 = 3^2 \rightarrow t^2 + 2 + \frac{1}{t^2} = 9 \rightarrow t^2 + \frac{1}{t^2} = 7$. So, $\left(t + \frac{1}{t}\right)^3 = \left(t^2 + \frac{1}{t^2}\right)\left(t + \frac{1}{t}\right) = 7(3) \rightarrow t^3 + t + \frac{1}{t} + \frac{1}{t^3} = 21 \rightarrow t^3 + 3 + \frac{1}{t^3} = 21 \rightarrow t^3 + \frac{1}{t^3} = 18$. Finally, $\left(t^2 + \frac{1}{t^2}\right)\left(t^3 + \frac{1}{t^3}\right) = 7(18) \rightarrow t^5 + t + \frac{1}{t} + \frac{1}{t^5} = 126 \rightarrow t^5 + 3 + \frac{1}{t^5} = 126 \rightarrow t^5 + \frac{1}{t^5} = \mathbf{123}$. Alternatively, using the binomial theorem yields $27 = 3^3 = \left(t + \frac{1}{t}\right)^3 = t^3 + 3\left(t + \frac{1}{t}\right) + \frac{1}{t^3} = t^3 + 9 + \frac{1}{t^3}$, so $t^3 + \frac{1}{t^3} = 27 - 9 = 18$. Also, $243 = 3^5 = \left(t + \frac{1}{t}\right)^5 = t^5 + 5t^3 + 10\left(t + \frac{1}{t}\right) + 5\frac{1}{t^3} + \frac{1}{t^5}$. Thus, $t^5 + \frac{1}{t^5} = 243 - 5\left(t^3 + \frac{1}{t^3}\right) - 10\left(t + \frac{1}{t}\right) = 243 - 5 \times 18 - 10 \times 3 = \mathbf{123}$.

Sprint 27

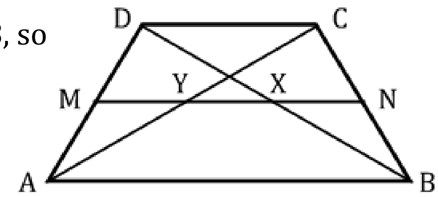
In base b, let $r = 0.5757\ldots$. So, $b^2r = 57.5757\ldots$. Now, $b^2r - r = 57 \rightarrow r(b^2 - 1) = 57 \rightarrow r = \frac{57}{b^2-1} = \frac{5b+7}{b^2-1}$, and $3r = 3\left(\frac{5b+7}{b^2-1}\right) = \frac{15b+21}{b^2-1}$. Also in base b, let $3r = 1.0606\ldots$. Similarly, $3r = 1 + \frac{6}{b^2-1}$. Now, setting these two expressions for $3r$ equal to each other gives us $\frac{15b+21}{b^2-1} = 1 + \frac{6}{b^2-1}$. Simplifying and solving for b, we get $\frac{b^2+5}{b^2-1} = \frac{15b+21}{b^2-1} \rightarrow b^2 + 5 = 15b + 21 \rightarrow b^2 - 15b - 16 = 0$. Factoring the left side of the equation, we see that $(b - 16)(b + 1) = 0$, so $b = 16$ or $b = -1$. Since a number base must be greater than 1, we have $b = 16$. So, $r = \frac{5(16)+7}{16^2-1} = \frac{87}{255} = \mathbf{\frac{29}{85}}$.

Sprint 28

In the interior of the figure are 2 circles, each of which is shared among 3 hexagons. There are 4 circles at the indentations of the outer boundary that are each shared by 2 hexagons. Each of the 10 remaining circles is dedicated to one particular hexagon. If we add up the sums for all four hexagons, we get $4S$, which includes 3 copies of the values in 2 circles and 2 copies in 4 circles. To maximize S, we need to maximize $4S$. The greatest value we can hope for is to put the greatest integers in the most used circles, which would mean 16 and 15 used three times each in the interior circles, and 14, 13, 12, and 11 used twice each in the circles at the boundary indentations. That would mean $4S = 3(16 + 15) + 2(14 + 13 + 12 + 11) + (10 + 9 + \cdots + 2 + 1) = 93 + 100 + 55 = 248$. So, $S = 62$. This figure shows how each number from 1 through 16 can be used once, and the sum around each hexagon is **62**.



Sprint 29



Trapezoid ABCD represents the dartboard. $\overrightarrow{AY}$ is the bisector of ∠DAB, so that points below the ray are closer to side AB of the trapezoid and points above the segment are closer to side DA. [Note that the figure appears to show that $\overrightarrow{AY}$ passes through C, and that is, indeed, the case, but that is merely a coincidence based on the dimensions of this trapezoid.] Similarly, $\overrightarrow{BX}$ is the bisector of ∠ABC, as the demarcation of points closer to side AB versus side BC. $\overline{\text{MN}}$ is the median of trapezoid ABCD and is the demarcation of points closer to side AB versus side CD. Thus, ABXY forms a trapezoid enclosing the points of trapezoid ABCD that are at least as close to side AB as to any other side of ABCD, which means we need to determine the ratio of the enclosed area of ABXY to the enclosed area of ABCD. Trapezoid ABCD can be partitioned into a rectangular region of width 12 in the middle and two congruent right triangles, one on each end with width 6; all the regions have the same, as yet unknown, height. The hypotenuse of the right triangles is given as 12 and the base leg as 6, so the altitude leg is $\sqrt{12^2 - 6^2} = \sqrt{108} = 6\sqrt{3}$; this is the height of trapezoid ABCD, and half that, or $3\sqrt{3}$, is the height of trapezoid ABXY. Also, because the base leg of each right triangle is 1/2 of the hypotenuse, we know that angles DAB and ABC have measure 60°, so angles YAB and ABX have measure 30°. The longer leg of a 30°-60°-90° triangle is $\sqrt{3}$ times as long as the shorter leg, so the "run" of a 30° slope is $\sqrt{3}$ times the "rise"; we found the rise to be $3\sqrt{3}$, so the run is $\sqrt{3} \times 3\sqrt{3} = 9$. Thus, $\text{YX} = \text{AB} - 2 \times 9 = 6$. As a result, the width of the median of ABXY is $\frac{24+6}{2} = 15$ while the width of the median of ABCD is $\frac{24+12}{2} = 18$. Thus, the desired ratio of the two areas is $\frac{15 \times 3\sqrt{3}}{18 \times 6\sqrt{3}} = \frac{5}{6} \times \frac{1}{2} = \mathbf{\frac{5}{12}}$.

Sprint 30

Let $x = \frac{1}{2}$. Then we have $f\left(1 - \frac{1}{2}\right) = 1 - f\left(\frac{1}{2}\right)$, so $f\left(\frac{1}{2}\right) = \frac{1}{2}$. Thus, $f\left(\frac{1}{3}\right) = f\left(\frac{\frac{1}{2}}{\frac{1}{2}+1}\right) = \frac{f\left(\frac{1}{2}\right)}{2} = \frac{1}{4}$. Now, $f\left(\frac{2}{3}\right) = 1 - f\left(\frac{1}{3}\right) = 1 - \frac{1}{4} = \frac{3}{4}$, which is the first term of the sum. For all positive integers n, $\frac{\frac{2}{2n-1}}{\frac{2}{2n-1}+1} = \frac{2/(2n-1)}{(2n+1)/(2n-1)} = \frac{2}{2n+1}$, so $f\left(\frac{2}{2n+1}\right) = \frac{1}{2} f\left(\frac{2}{2n-1}\right)$. This means each successive term is $\frac{1}{2}$ of its predecessor, so we have an infinite geometric series whose first term is $\frac{3}{4}$ and the common ratio is $\frac{1}{2}$, so the sum is $\frac{\frac{3}{4}}{1-\frac{1}{2}} = \mathbf{\frac{3}{2}}$.

Target 1
The sum of products is greatest when greater values for the variable go with greater numeric coefficients, so $817 \times 3 + 512 \times 2 + 210 \times 1 = 2451 + 1024 + 210 = \mathbf{3685}$.

Target 2
The total amount of popcorn kernels Karen had was $14 \times \left(6 \text{ lb} + 9\frac{5}{8} \text{ oz}\right) + \left(4 \text{ lb} + 7\frac{3}{4} \text{ oz}\right) =$
$84 \text{ lb} + \left(126 + \frac{35}{4}\right) \text{ oz} + 4 \text{ lb} + 7\frac{3}{4} \text{ oz} = 88 \text{ lb} + 142.5 \text{ oz} = 88 \text{ lb} + 8 \text{ lb} + 14.5 \text{ oz} = 96 \text{ lb} + 14.5 \text{ oz}$.
So, $x = 96$, $y = 14.5$ and $x + y = 96 + 14.5 = \mathbf{110.5}$.

Target 3
Start with 48, with each of the following $12 - 8 = 4$ years afterward bearing 1.5 times as many as the previous year. Thus, that last year will yield $48 \times 1.5^4 = \mathbf{243}$ apples.

Target 4
The triangle is a right isosceles triangle. Thus, angles B and C measure 45°, and each of the arcs centered at B and C is 1/8 of a circle; being the same radius, the two arcs can be combined to 1/4 of a circle, like the arc centered at A. The radius of the arc centered at A is the altitude to $\overline{\text{BC}}$ from A, thus $3\sqrt{2}$ cm, so that the radius of the smaller arcs is $6 - 3\sqrt{2}$ cm. The area of the unshaded region is $\frac{1}{4}\pi\left(\left(3\sqrt{2}\right)^2 + \left(6 - 3\sqrt{2}\right)^2\right) = \frac{\pi}{4}\left(18 + 36 - 36\sqrt{2} + 18\right) = \pi\left(18 - 9\sqrt{2}\right) \text{ cm}^2$. The total area enclosed by the triangle is $\frac{1}{2}(6)^2 = 18 \text{ cm}^2$. So, the shaded region has area $18 - \pi\left(18 - 9\sqrt{2}\right) \approx$ $\mathbf{1.44} \text{ cm}^2$.

Target 5
Let the sequence be 1, c, A, 6, d, e, 7, B, f, 12, g, h, 13. Following 12, g must be 9, 13, or 14; 13 is already used and 14 is out of range, so $g = 9$. Getting from 9 to 13 in two steps requires $h = 11$. Going from 7 to 12 in three steps requires $7 < B \le 9 < 10 \le f < 12$; 9 is already used, so $B = 8$ and $f = 10$. This leaves only 2, 3, 4 and 5 to be chosen. Thus, d must be a drop of 3 from 6, so $d = 3$ and $e = 5$. That means $c = 2$ and $A = 4$. Thus, $AB = 4 \times 8 = \mathbf{32}$.

Target 6
The area enclosed by △ADM is the area enclosed by △ABD minus the area enclosed by △ABM. CD = CF − EF + ED, so $EF = CF + ED - CD = 8 \text{ ft} + 8 \text{ ft} - 14 \text{ ft} = 2 \text{ ft}$. That means the median of trapezoid ABEF is $\frac{8+2}{2} = 5$ ft, which, combined with a total area of 60 ft², means the height is 12 ft. $AB = 8$ ft serves as a common base for triangles ABD and ABM. The height of △ABD is the height of the trapezoid, 12 ft. Triangles ABM and CDM are similar with a linear ratio of 8 ft :14 ft, thus 4:7. The sum of the altitudes from M to $\overline{\text{AB}}$ and to $\overline{\text{EF}}$ is the 12-ft height of the trapezoid, so the height of △ABM is $\frac{4}{4+7} \times 12$ ft. Finally, the difference in the areas enclosed by △ABD minus △ADM, with the common base of 8 ft, is $\frac{1}{2}(8 \text{ ft})\left(\left(1 - \frac{4}{11}\right)(12 \text{ ft})\right) = \mathbf{\frac{336}{11}} \text{ ft}^2$.

Target 7

The points of intersection of $y = x^2 - x - 2$ and $y = x + 1$ occur where the two expressions for y are equal. So, $x^2 - x - 2 = x + 1 \rightarrow x^2 - 2x - 3 = 0 \rightarrow (x + 1)(x - 3) = 0$. So, the points of intersection occur when $x = -1$ and $x = 3$. Substituting for x in the equation $y = x + 1$, we see that when $x = -1$, $y = -1 + 1 = 0$, and when $x = 3$, $y = 3 + 1 = 4$. Thus, the points of intersection are $(-1, 0)$ and $(3, 4)$, and the distance between these two points is $\sqrt{\left(3 - (-1)\right)^2 + (4 - 0)^2} = \mathbf{4\sqrt{2}}$.

Target 8

For the first transformation, any of the 11 options will achieve a satisfactory result (none of the 6 points has yet been transformed to). After that, the remaining 5 points can be transformed to in any order, as long as there is no repetition, so $5! = 120$ orders for visiting points. At each step after the first, there are two transformations (one rotation and one reflection) that will get from one point to the next [distinct] point. Thus, the overall probability of success is $120 \times \frac{11}{11} \times \frac{2}{11} \times \frac{2}{11} \times \frac{2}{11} \times \frac{2}{11} \times \frac{2}{11} = \frac{42{,}240}{11^6}$, so $k = \mathbf{42{,}240}$.