

MATHCOUNTS®

2021 CHAPTER INVITATIONAL COMPETITION Sprint Round Problems 1–30

Name _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.



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1. _____ What is the arithmetic mean of the terms in the arithmetic sequence 12, 18, 24, 30, 36?

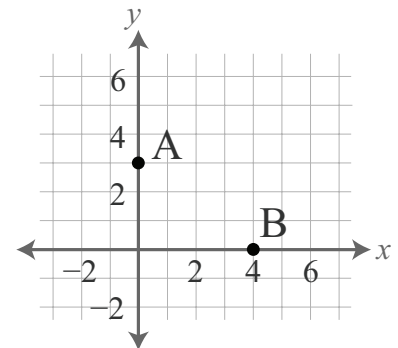
2. _____ squares If Skylar colors $\frac{1}{2}$ of $\frac{2}{3}$ of the squares in this figure, how many squares does he color?



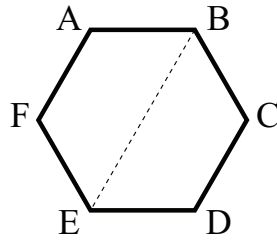
3. _____ What is the median of the first seven prime numbers?

4. _____ What is the absolute difference between five less than a number n and seven more than n ?

5. _____ units What is the distance between points A and B on the coordinate grid shown?



6. _____ units² Regular hexagon ABCDEF, shown here, has area 5 units². What is the area of trapezoid BCDE? Express your answer as a decimal to the nearest tenth.



7. _____ What is the value of the expression $\sqrt{9^2 - 5 \times 4 + 12 \div 4}$?

8. _____ Given that $3a + 5 = 7b - 11$ and $a = b$, what is the value of a ?

9. \$ _____ An album with twelve songs costs \$9.99. If Andy buys each song individually, he pays \$0.99 per song. How much money can Andy save by buying the album instead of each individual song?

10. _____ What is the integer value of $\text{👤} + \text{🍦} + \text{🐱}$, given these three equations?

$$\text{👤} + \text{👤} + \text{👤} = 90$$

$$\text{👤} + \text{🍦} + \text{🍦} = 60$$

$$\text{👤} + \text{👤} - \text{🐱} = 40$$

11. _____ hours At Alicia's new job, she works three days a week for a total of 45 hours per week. At her old job, she worked four days a week for a total of 40 hours per week. What is the absolute difference in the average number of hours Alicia worked per workday at her old and new jobs?

12. _____ What is the value of x if $x^x = 256$?

13. _____ What is the least positive integer that has five distinct prime factors?

14. _____ $\frac{\text{beats per}}{\text{minute}}$ Jasmine estimates her maximum heart rate, in beats per minute, by subtracting her age from 220. Jasmine's heart rate during exercise should be between 50% and 85% of her maximum heart rate. If Jasmine is 20 years old, what is the highest heart rate, in beats per minute, that she should have during exercise? Express your answer to the nearest whole number.

15. _____ The arithmetic mean of three distinct integers is 11, and the range of the three integers is 2. What is the smallest of these integers?

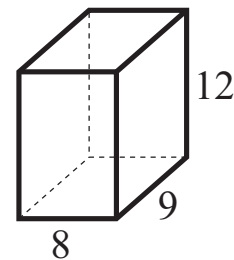
16. _____ Let $a \& b = ab + b + a - 1$. If $a \& 7 = 70$, what is $a \& 3$?

17. _____ : _____ a.m. Kody's alarm rings every 11 minutes, and his dad calls upstairs every 15 minutes to wake him up. But Kody will only wake up if his alarm rings and his dad calls him at the same time. If his alarm first rings at 7:00 a.m. and his dad first calls him at 7:05 a.m., at what time will Kody wake up?

18. _____ inches The largest rectangular region shown is partitioned into three rectangular regions labeled A, B and C. The lengths, in inches, of rectangles A, B and C are $n + 3$, $n + 5$ and $n - 2$, respectively, and all three rectangles have width n . If the area of rectangle B is 84 in^2 , what is the perimeter of the largest rectangle shown?



19. _____ cm What is the length of the longest diagonal of the rectangular prism shown, which has length 9 cm, width 8 cm and height 12 cm?



20. _____ % Suppose there is an 80% chance of rain each day. On days that it rains, Kathy has a 45% chance of being late, compared to a 30% chance when it does not rain. On a random day, what is the probability that Kathy will be late?

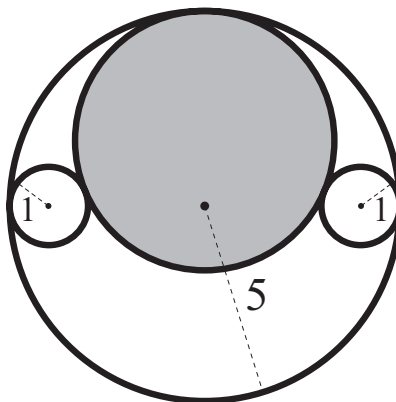
21. _____ inches Triangle ABC has sides of length 11 inches, 15 inches and 16 inches. What is the length of the altitude to the side of length 15 inches? Express your answer in simplest radical form.

22. _____ boxes Julie is packing marbles into boxes. She needs to pack away 815 marbles. She has boxes that can hold 10, 25, 50 or 100 marbles. If Julie can use at most 5 boxes of each size and must fill each box she uses, what is the minimum number of boxes she needs to pack all her marbles?

23. _____ What positive number x has the property that $\sqrt[6]{x^7} - 6\sqrt[3]{x^2} = 0$?

24. _____ What fraction is equivalent to $0.7\overline{12}$? Express your answer as a common fraction.

25. _____ inches In the figure, two circles of radius 1 inch are internally tangent to a circle of radius 5 inches so that the centers of all three circles are collinear. The shaded fourth circle is tangent to each of the other three circles as shown. What is the radius of the shaded circle? Express your answer as a common fraction.

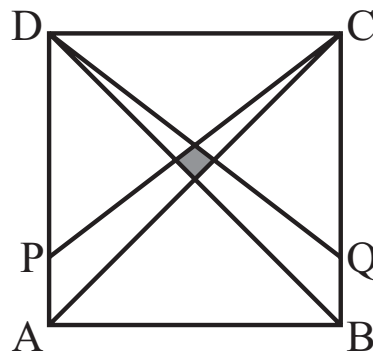


26. _____ The greatest of three nonnegative integers is one more than twice the sum of the other two integers. The arithmetic mean of the three integers, rounded to the nearest integer, happens to equal the median of the set. What is the product of the three integers?

27. _____ Suppose $p(x)$ is a polynomial such that $p(x) = p(1) + p(2) \cdot x + x^2$ for all numbers x . What is the value of $p(5)$?

28. _____ Let $[n]$ be defined as the greatest integer less than or equal to n . What is the value of n if $[n] \times n = 3$? Express your answer as a decimal to the nearest tenth.

29. _____ units^2 Square ABCD, shown here, has side length 6 units. Points P and Q are located on sides AD and BC, respectively, with $AP = BQ = 1$ unit. Triangles ACP and BDQ overlap in the square to form the shaded quadrilateral. What is the area of the shaded quadrilateral? Express your answer as a common fraction.



30. _____ $\frac{\text{ordered}}{\text{triples}}$ How many ordered triples of positive integers (a, b, c) have $\text{GCF}(a, b, c) = 2020$ and $\text{LCM}(a, b, c) = 2020^2$?